Optimal Distillation of Coherent States

with Phase-Insensitive Operations

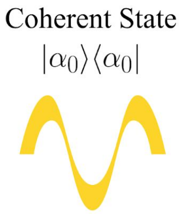
Shiv Akshar Yadavalli
Quantum Resources Workshop 2026

arXiv:2409.05974
SAY & Iman Marvian

Coherent States

- Model "classical-like" behavior of EM waves in photon modes

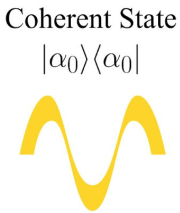
- Minimize Heisenberg uncertainty

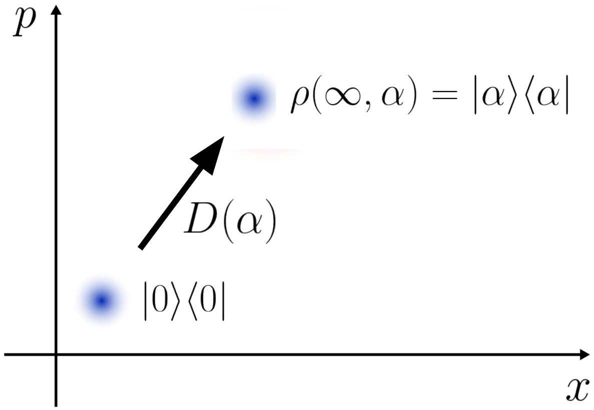


Coherent States

- Model "classical-like" behavior of EM waves in photon modes

- Minimize Heisenberg uncertainty



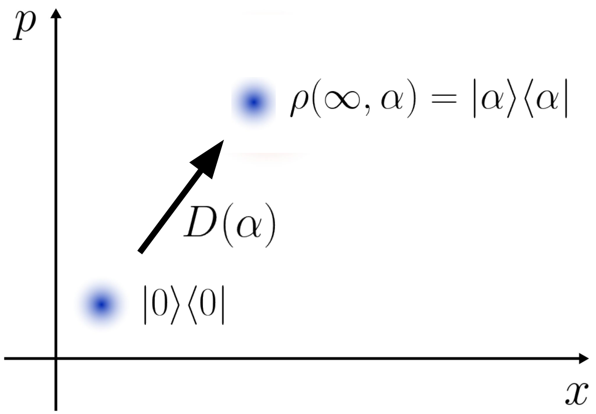


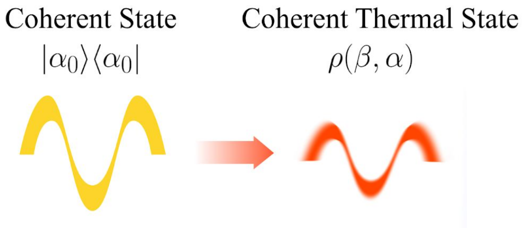
Poisson profile in the Fock (occupation number) basis:

$$|\alpha\rangle = D(\alpha)|0\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \quad \alpha \in \mathbb{C}$$

Coherent States

- Model "classical-like" behavior of EM waves in photon modes
- Minimize Heisenberg uncertainty



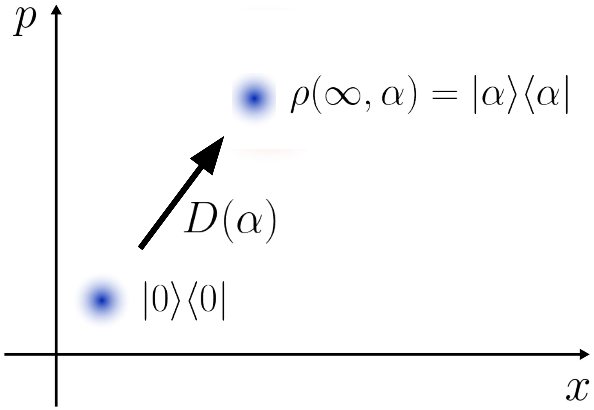


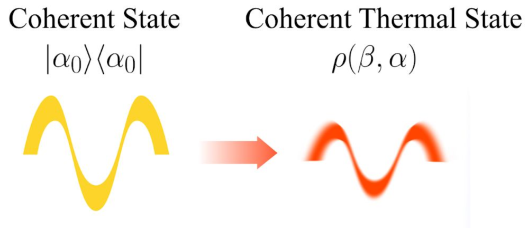
Coherent Thermal States

- Noisy output of a thermal attenuator channel
- Driving a thermal state rather than the vacuum state (zero temp.)

Coherent States

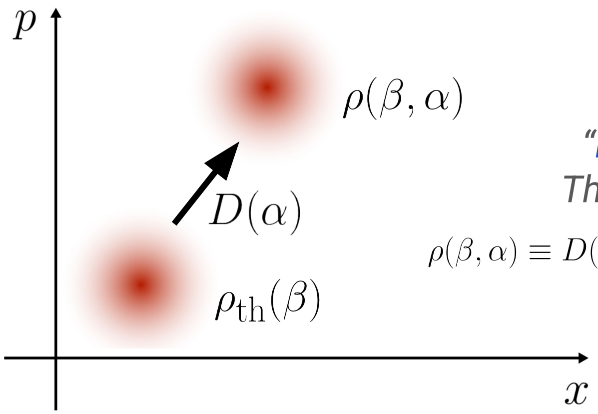
- Model "classical-like" behavior of EM waves in photon modes
- Minimize Heisenberg uncertainty





Coherent Thermal States

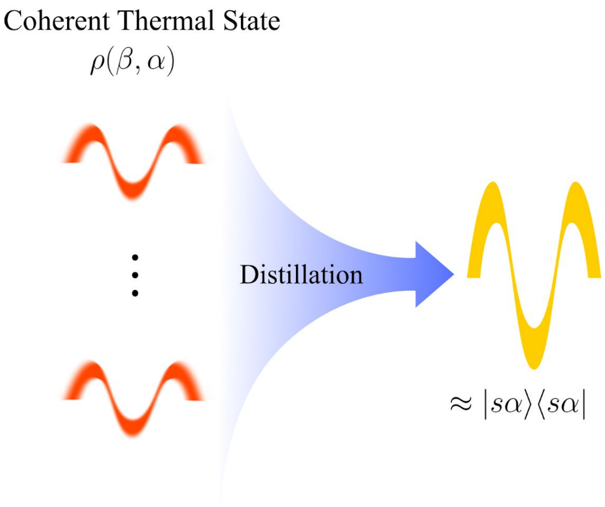
- Noisy output of a thermal attenuator channel
- Driving a thermal state rather than the vacuum state (zero temp.)



"Displaced" Thermal State

$$\rho(\beta, \alpha) \equiv D(\alpha)\rho_{\text{th}}(\beta)D^{\dagger}(\alpha)$$

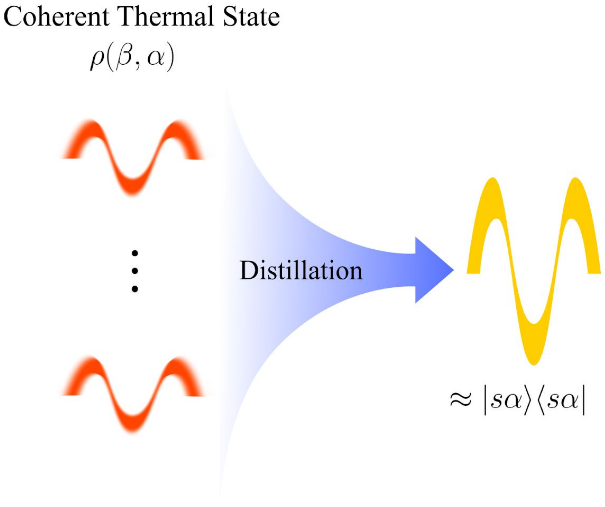
Distillation of Coherent States



From many copies of $\rho(\beta,\alpha)$, how well can we distill a single pure coherent state?

$$\rho(\beta, \alpha)^{\otimes n} \longrightarrow |\alpha\rangle\langle\alpha|$$

Distillation of Coherent States



From many copies of $\rho(\beta,\alpha)$, how well can we distill a single pure coherent state?

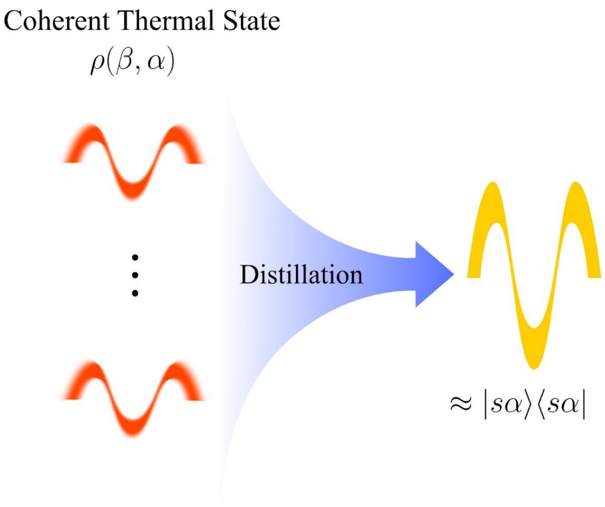
$$\rho(\beta, \alpha)^{\otimes n} \longrightarrow |\alpha\rangle\langle\alpha|$$

Bosonic State Preparation:

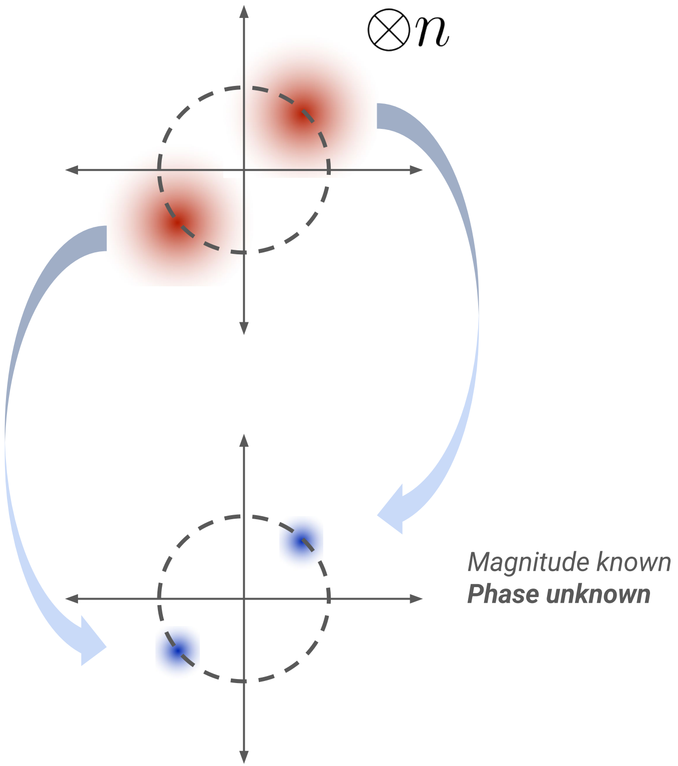
State cooling

Distillation lowers temperature

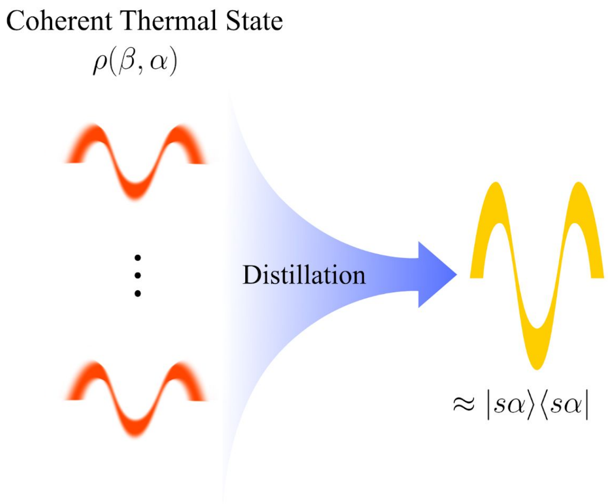
Distillation of Coherent States *with* Phase-insensitive Operations

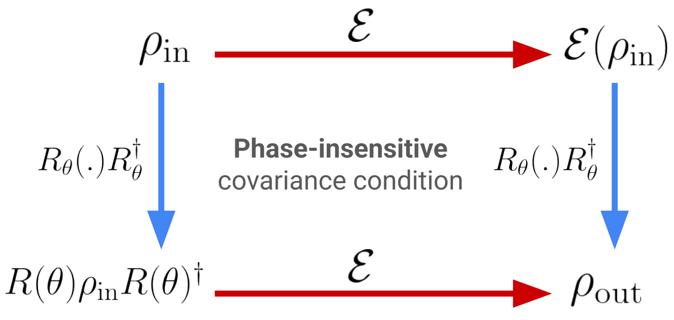


Input & Output phase are ***in-sync***
(Essential in, e.g., interferometry)



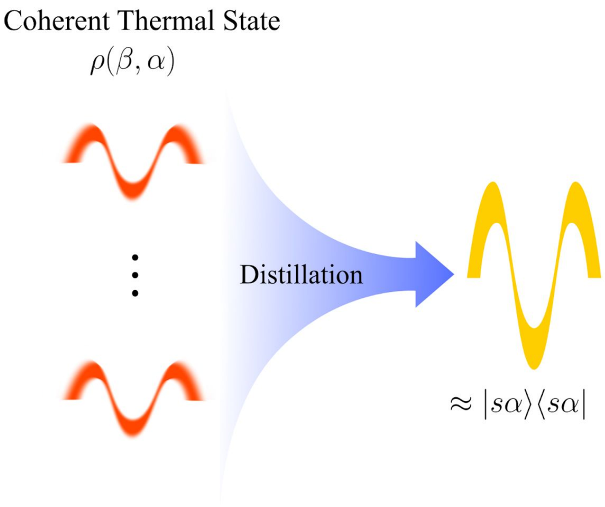
Distillation of Coherent States *with* Phase-insensitive Operations



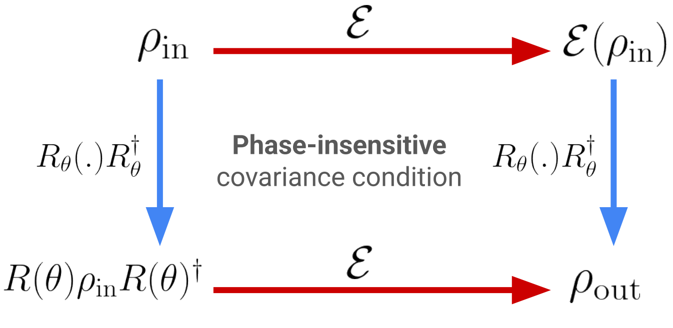


Input & Output phase are ***in-sync***
(Essential in, e.g., interferometry)

Distillation of Coherent States *with* Phase-insensitive Operations



Input & Output phase are ***in-sync***
(Essential in, e.g., interferometry)



From many copies of $\rho(\beta,\alpha)$, how well can we distill a single pure coherent state, with *phase-insensitive* operations?

$$\mathcal{E}_n : \rho(\beta, \alpha)^{\otimes n} \to |\alpha\rangle\langle\alpha|$$

E.g., phase-insensitive amplifiers,
interferometry

Zoom out…

- Example of *State transformations* in the **Resource Theory of Asymmetry**
  - Specifically, the Resource Theory of *U(1)* Asymmetry

Zoom out…

- Example of *State transformations* in the **Resource Theory of Asymmetry**
  - Specifically, the Resource Theory of *U(1)* Asymmetry
- Equivalently, the Resource Theory of *Time Translation* Asymmetry

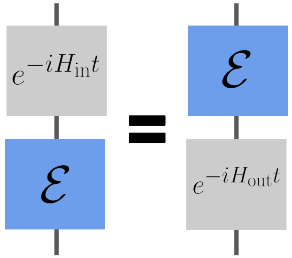
Fix Hamiltonians for input/output systems: $H_{\mathrm{in}}, H_{\mathrm{out}}$

Free States

$$[\rho, H] = 0$$

Stationary States

Free Operations



Time-Translation Invariant Ops.

Zoom out…

- Example of *State transformations* in the **Resource Theory of Asymmetry**
  - Specifically, the Resource Theory of *U(1)* Asymmetry
- Equivalently, the Resource Theory of *Time-Translation* Asymmetry
  - *Resource Theory of Coherence : Coherence Distillation*

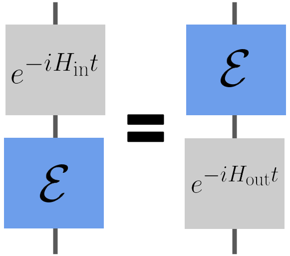
Fix Hamiltonians for input/output systems: $H_{\text{in}}, H_{\text{out}}$

Free States

$$[\rho, H] = 0$$

Stationary States

Free Operations



Time-Translation Invariant Operations **=** ***Completely*** *Incoherence-Preserving Operations*

Resource States

$$[\rho, H] \neq 0$$

'Coherent' States
States w/ Coherence

Ultimate Limits of Coherence Distillation

$$\mathcal{E}_n : \rho(t)^{\otimes n} \mapsto |\psi(t)\rangle\langle\psi(t)|$$

Time-Translation *U(1)* Invariant
Distillation Protocol

Ultimate Limits of Coherence Distillation

$$\mathcal{E}_n : \rho(t)^{\otimes n} \mapsto |\psi(t)\rangle\langle\psi(t)|$$

Time-Translation *U(1)* Invariant Distillation Protocol

Benchmarking Distillation Protocols:

$$\langle\psi(t)|\mathcal{E}_n(\rho(t)^{\otimes n})|\psi(t)\rangle = 1 - \frac{\delta_{\mathcal{E}}}{n} + o\left(\frac{1}{n}\right)$$

Linear coefficient:
'Infidelity Factor'

Ultimate Limits of Coherence Distillation

$$\mathcal{E}_n : \rho(t)^{\otimes n} \mapsto |\psi(t)\rangle\langle\psi(t)|$$

Time-Translation *U(1)* Invariant Distillation Protocol

Benchmarking Distillation Protocols:

$$\langle\psi(t)|\mathcal{E}_n(\rho(t)^{\otimes n})|\psi(t)\rangle = 1 - \frac{\delta_{\mathcal{E}}}{n} + o\left(\frac{1}{n}\right)$$

Universal Lowerbound from Monotonicity

$$\delta_{\mathcal{E}} \geq \frac{\mathrm{Var}_H(|\psi\rangle\langle\psi|)}{P_H(\rho)}$$

[Marvian 2020]

P_H (**Purity of Coherence**) is *resource monotone*

$$P_H(\rho) = -\mathrm{Tr}([H,\rho]^2\rho^{-1}) \quad \text{if } \sup(H\rho H) \subseteq \sup(\rho)$$

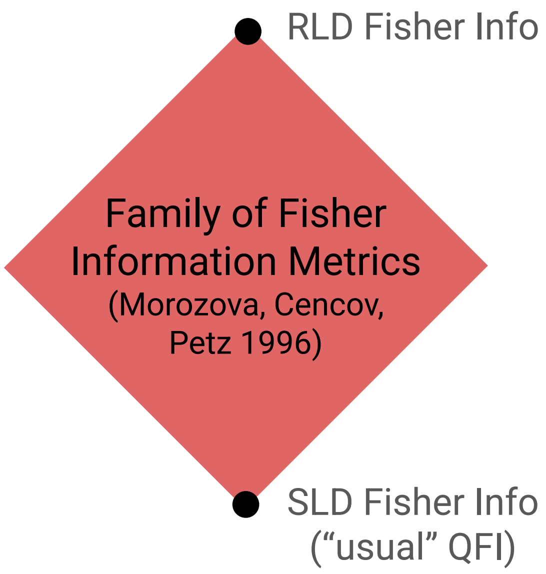
$$P_H(\rho) = \infty \quad \text{otherwise}$$

- *Infinite for Pure Coherent States*
- *Linear rate Coherence Distillation is impossible!*

Purity of Coherence from **Fisher Information Metrics**

Quantum Generalization of Classical Fisher Information Metric

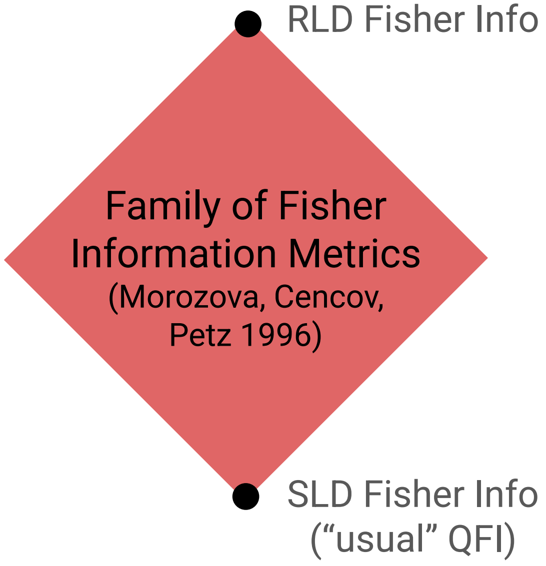
Metric on the convex space of density matrices

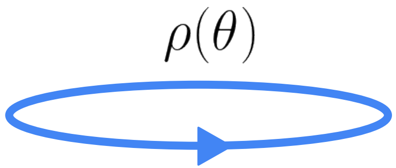


Purity of Coherence from **Fisher Information Metrics**

Quantum Generalization of Classical Fisher Information Metric

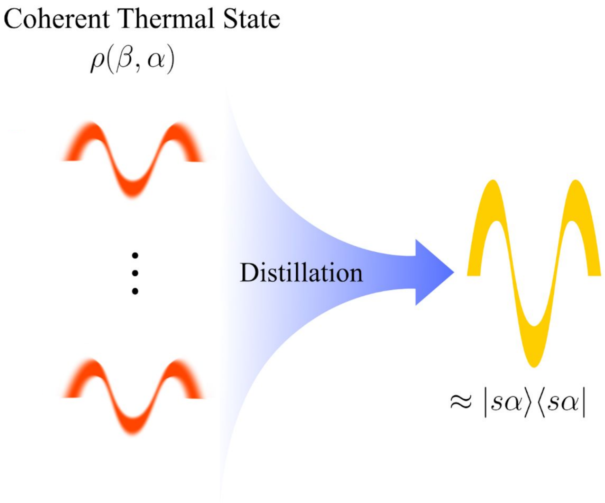
Metric on the convex space of density matrices





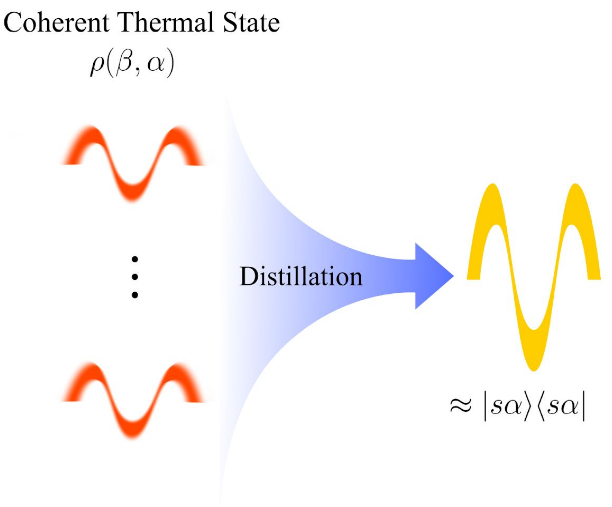
Purity of Coherence = RLD Fisher Info for the orbit of Hamiltonian evolution [Marvian 2020]

Distillation of Coherent States *with* Phase-insensitive Operations



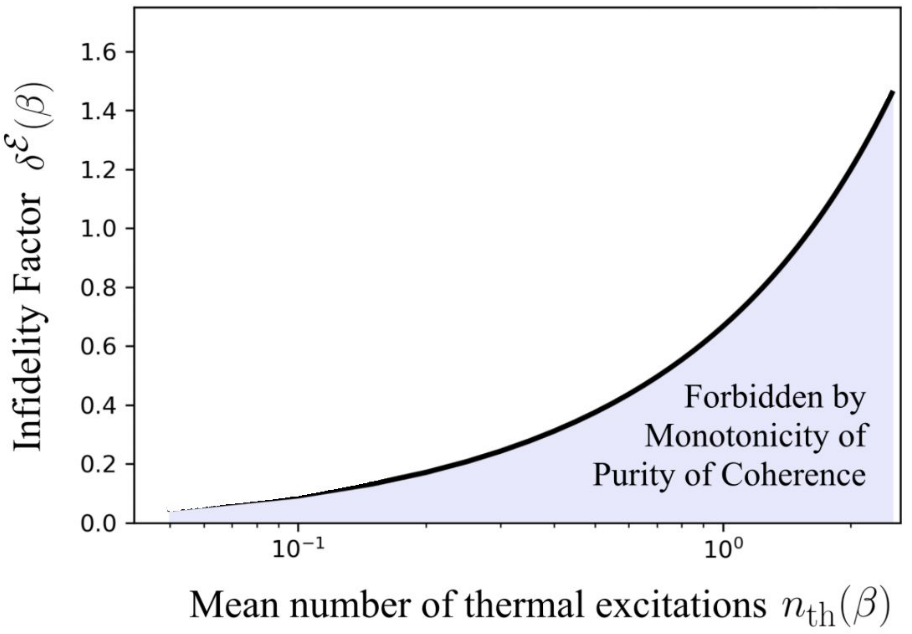
Coherence Distillation given
free Hamiltonian on H.O.'s

Distillation of Coherent States *with* Phase-insensitive Operations



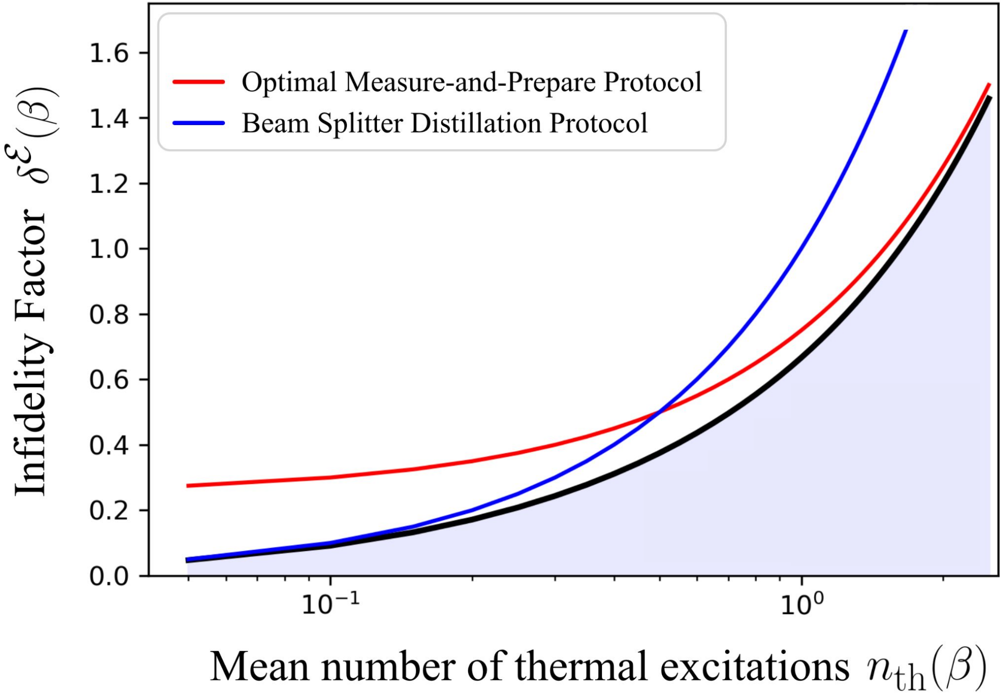
$$\delta_{\mathcal{E}} \geq \frac{\mathrm{Var}_H(|\psi\rangle\langle\psi|)}{P_H(\rho)}$$

$$\rho(\beta, \alpha)^{\otimes n} \longrightarrow |\alpha\rangle\langle\alpha|$$



Performance of Phase-Insensitive Distillation Protocols

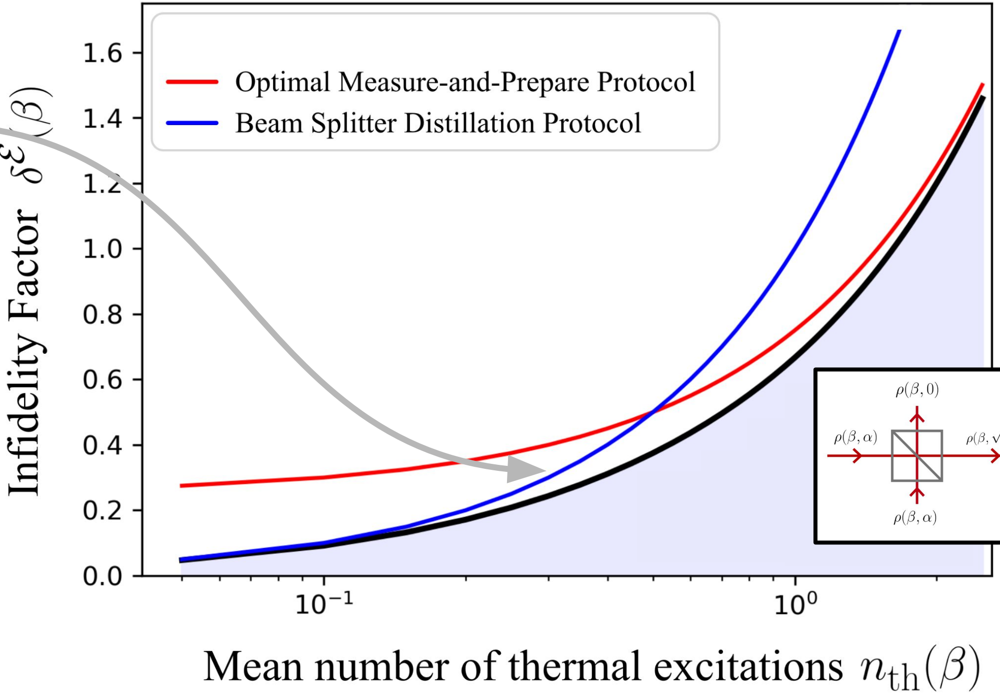
$$\rho(\beta, \alpha)^{\otimes n} \longrightarrow |\alpha\rangle\langle\alpha|$$



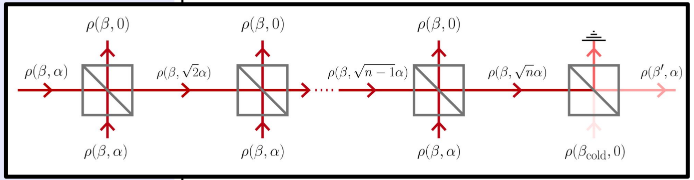
Performance of Phase-Insensitive Distillation Protocols

$$\rho(\beta,\alpha)^{\otimes n} \longrightarrow |\alpha\rangle\langle\alpha|$$

Optimal Gaussian phase-insensitive distillation

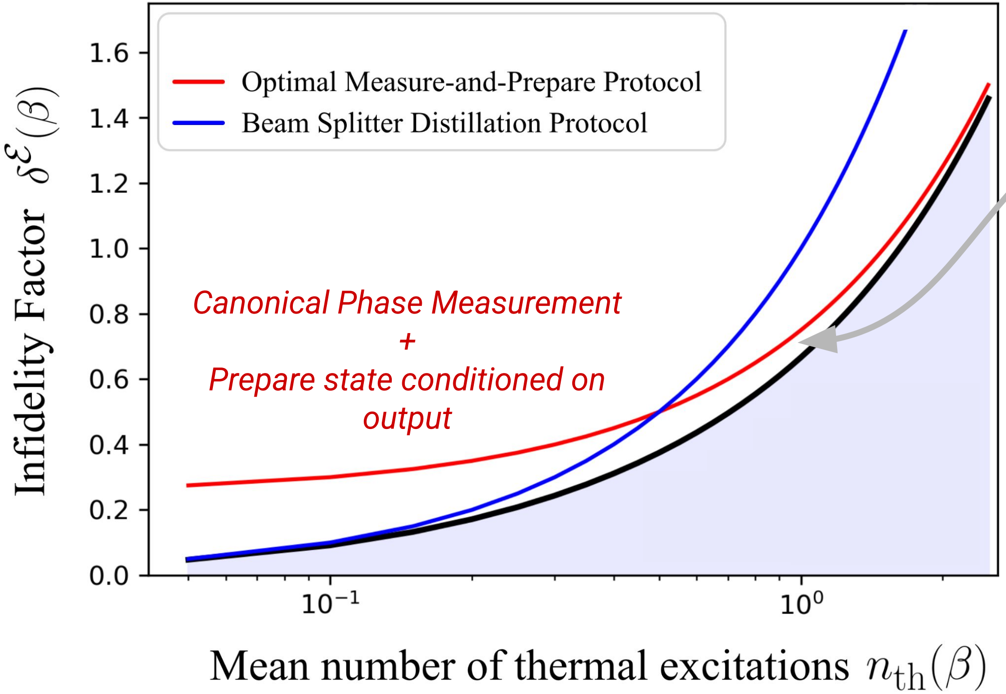


Realized by only ***linear optical*** *elements*
(passive operations)



Performance of Phase-Insensitive Distillation Protocols

$$\rho(\beta,\alpha)^{\otimes n} \longrightarrow |\alpha\rangle\langle\alpha|$$

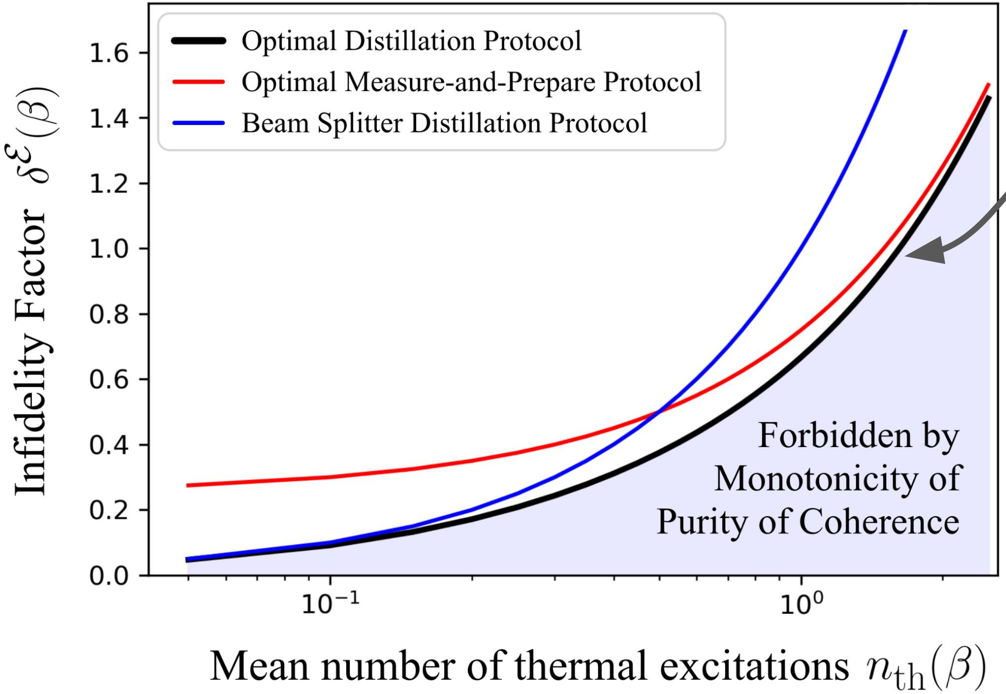


Optimal Measure-Prepare protocol

Governed by *SLD Fisher Information*

Lowerbound can be saturated!

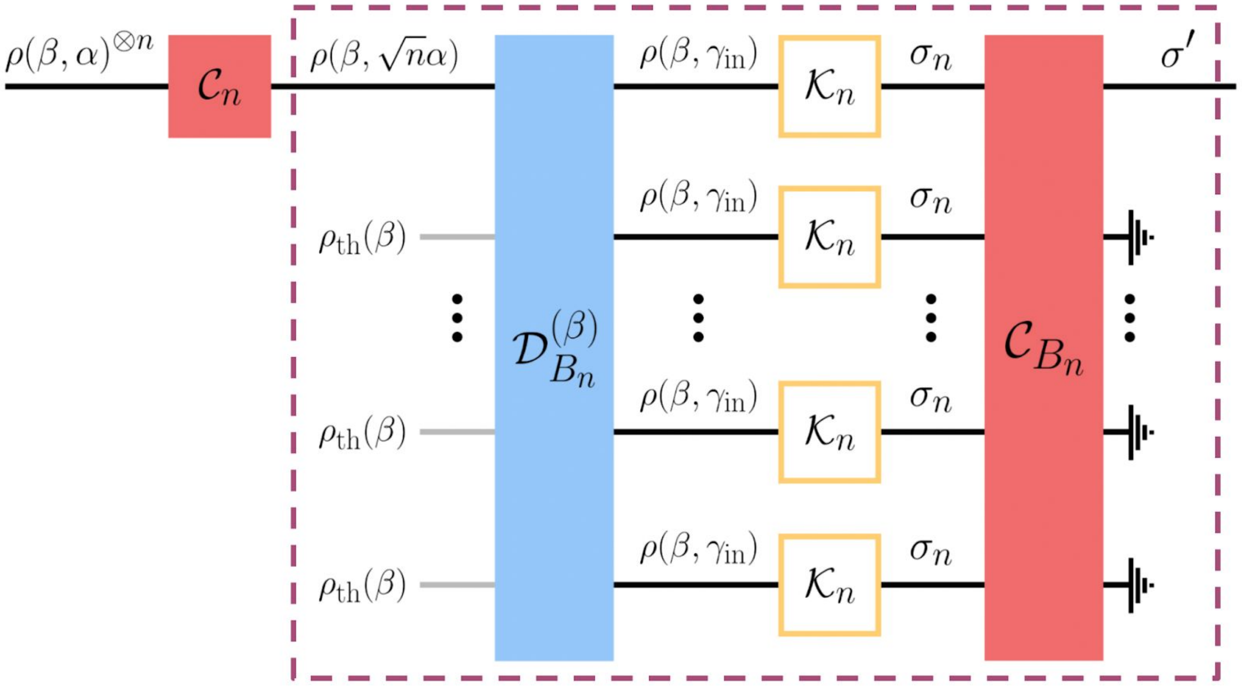
$$\rho(\beta, \alpha)^{\otimes n} \longrightarrow |\alpha\rangle\langle\alpha|$$



Main Result:
We construct an Optimal Phase-insensitive Distillation Protocol.
[arXiv:2409.05974]

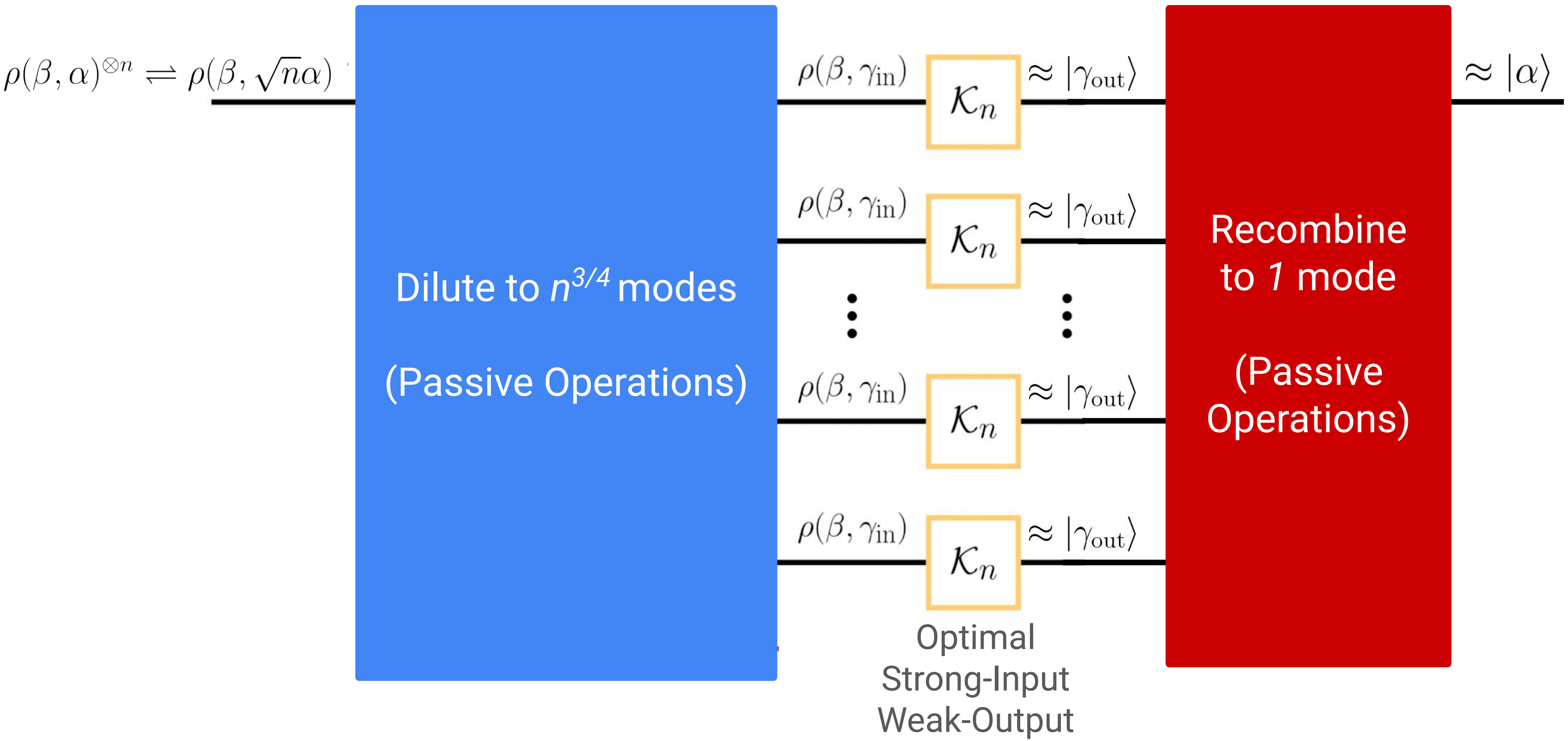
Endows Operational Meaning to RLD Fisher Info!

Optimal Phase-Insensitive Distillation Protocol

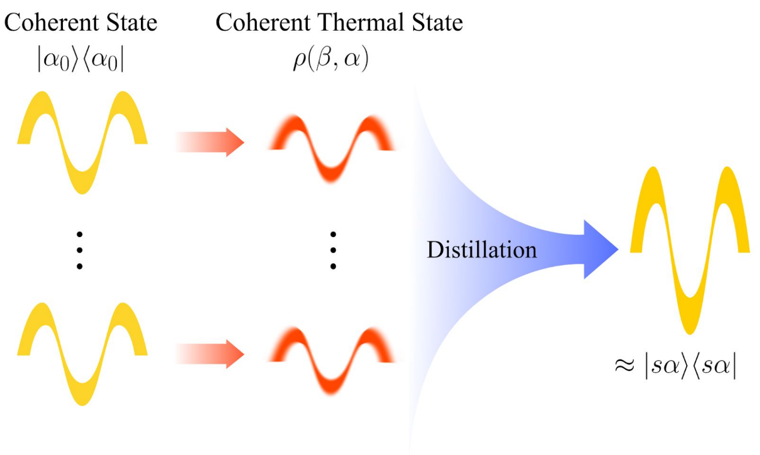


Non-Gaussian, Phase-insensitive & Optimal

Optimal Protocol



Summary



- We construct a novel optimal phase-insensitive protocol to distill coherent states from coherent thermal states.
- Optimal performance is determined by the Purity of Coherence: first operational interpretation of RLD Fisher Information metric

Invitation

Distillation is *generalization* of Metrology

Invitation

Distillation is *generalization* of Metrology

Task: Estimate phase θ from its noisy encoding ρ(θ)

$$\rho(\theta) \longrightarrow \theta$$

quantum encoding

classical estimate

Invitation

Distillation is *generalization* of Metrology

Task: *Distill* pure encoding of |θ> from its noisy encoding ρ(θ)

$$\rho(\theta) \longrightarrow \theta \quad |\theta\rangle\langle\theta|$$

quantum encoding

~~classical estimate~~

“quantum” estimate

Invitation

Distillation is *generalization* of Metrology

Task: *Distill* pure encoding of |θ> from its noisy encoding ρ(θ)

$$\rho(\theta) \longrightarrow \cancel{\theta} \quad |\theta\rangle\langle\theta|$$

quantum encoding — ~~classical estimate~~ — “quantum” estimate

Limits of *Metrology*:
SLD Fisher Information (QFI)

Limits of *Distillation*:
RLD Fisher Information
[qubits: arXiv 2510.08493] Poster by Sujay!

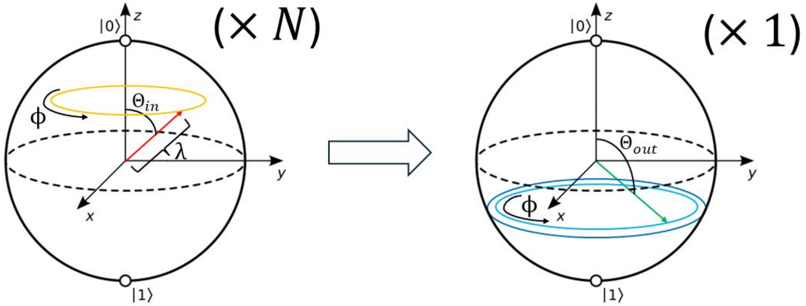
The analogous result also applies to qubits (Poster!!!)

Optimal Distillation of Qubit Clocks

Sujay Kazi[1,2] and Iman Marvian[1,2,3]

[1]*Department of Electrical and Computer Engineering, Duke University, Durham, NC 27708, USA*
[2]*Duke Quantum Center, Durham, NC 27708, USA*
[3]*Department of Physics, Duke University, Durham, NC 27708, USA*

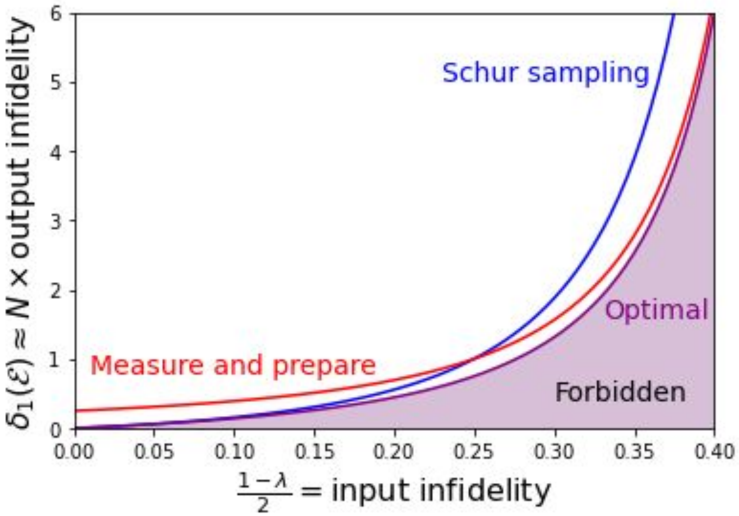
We study coherence distillation under time-translation-invariant operations: given many copies of a quantum state containing coherence in the energy eigenbasis, the aim is to produce a purer coherent state while respecting the time-translation symmetry. This symmetry ensures that the output remains synchronized with the input and



$$\mathcal{I}(\mathcal{E}_N) = \frac{\delta_1(\mathcal{E})}{N} + \frac{\delta_2(\mathcal{E})}{N^2} + \frac{\delta_3(\mathcal{E})}{N^3} + \cdots$$

$$\delta_1(\mathcal{E}) = \frac{1-\lambda^2}{4\lambda^2}\,\frac{\sin^2\Theta_{\text{out}}}{\sin^2\Theta_{\text{in}}}$$

Comes from RLD!



arXiv:2409.05974