QUANTUM RESOURCES 2026 – TOKYO, JAPAN

Nonlocality of quantum states can be transitive

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∃+∃+∃=∞

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QFort

QFort

Nonlocality as a resource

Device-independent certification

- Key distribution, verification of quantum devices

Intrinsic randomness and secrecy

- Randomness expansion, secret sharing

Distributed advantage

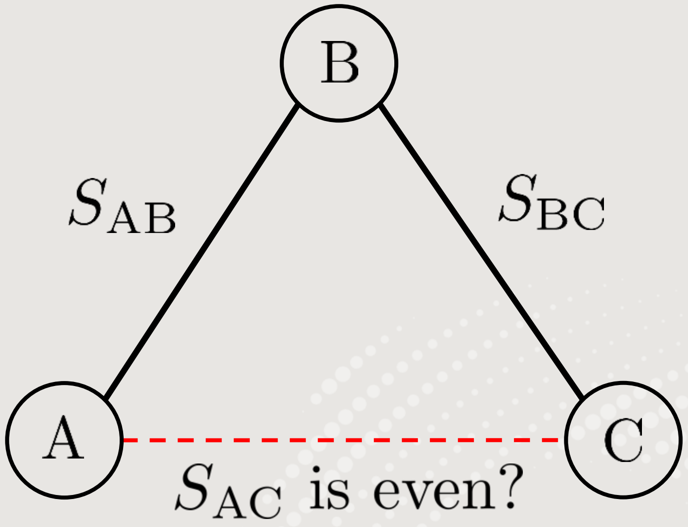
- Reduced communication complexity

Transitivity

Let A, B, C be unknown integers

- $S_{\mathrm{AB}} = A + B$ is even
- $S_{\mathrm{BC}} = B + C$ is even
- Is $S_{\mathrm{AC}} = A + C$ even?

Yes, S_{AC} is even



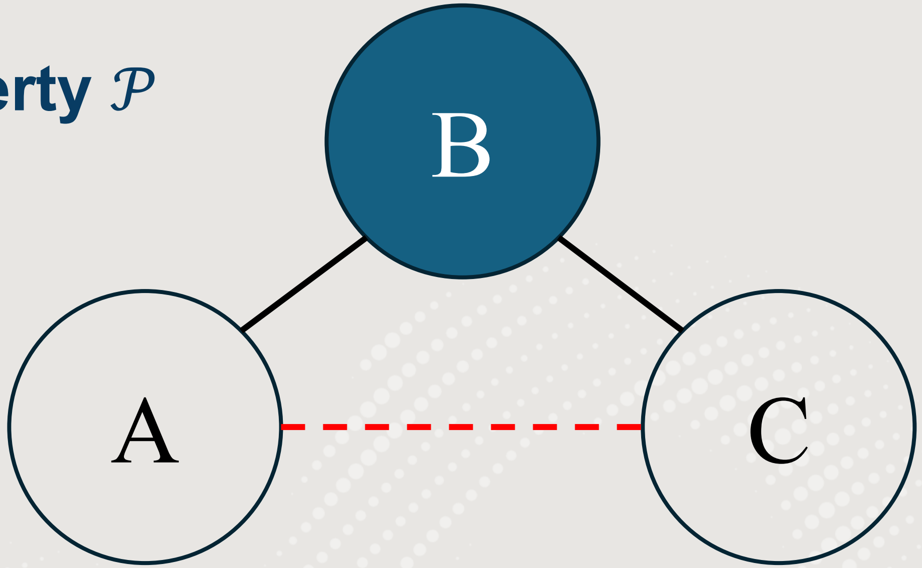
Fixing marginals can fully determine properties on other marginals

Transitivity

Quantum systems $\mathrm{A}, \mathrm{B}, \mathrm{C}$

- Let $\rho_{\mathrm{AB}}, \rho_{\mathrm{BC}}$ be given and have some property $\mathcal{P}$
- $\mathcal{P}$ can be entanglement, nonlocality, etc.

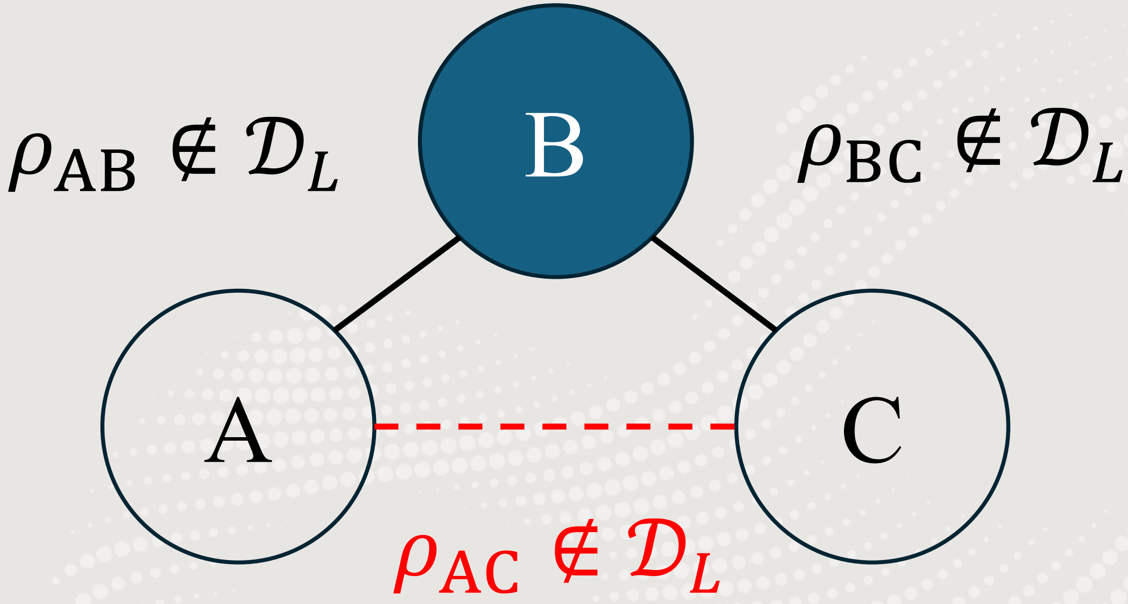
Does ρ_{AC} also have $\mathcal{P}$ for all ρ_{ABC}?



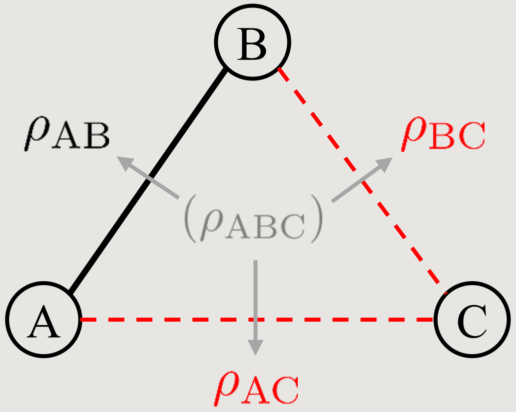
If yes, then $\rho_{\mathrm{AB}}, \rho_{\mathrm{BC}}$ exhibit $\mathcal{P}$-transitivity in AC

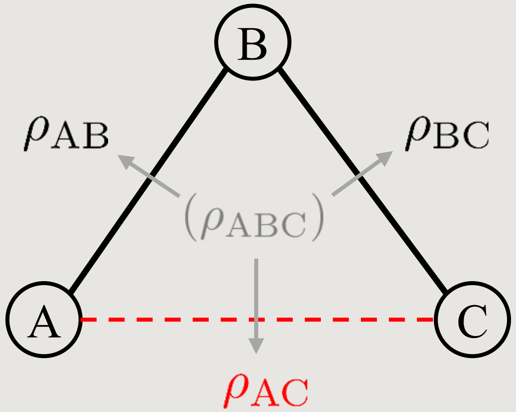
Main result

There exists nonlocal quantum states ρ_{AB} and ρ_{BC} such that for every compatible joint state ρ_{ABC}, ρ_{AC} is always nonlocal.



Monogamy vs. transitivity





Given entangled ρ_{AB}:

- $\forall$ compatible ρ_{ABC}
- $\rho_{BC}, \rho_{\mathrm{AC}}$ are separable

[V. Coffman *et al.*, Phys. Rev. A **61**.052306 (2000)]

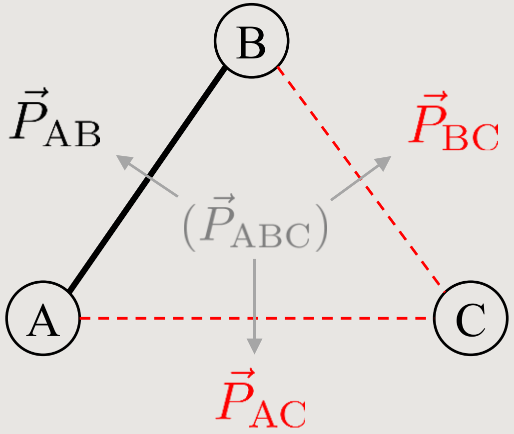
Given entangled $\rho_{\mathrm{AB}}, \rho_{\mathrm{BC}}$:

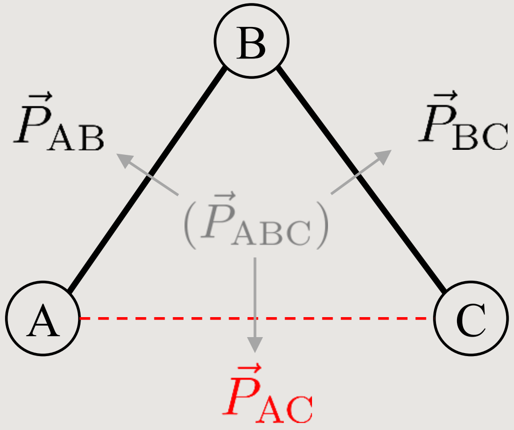
- $\forall$ compatible ρ_{ABC}
- ρ_{AC} is entangled

W state, Werner state, isotropic state,
Haar-random pure states, ...

[G. Tabia *et al.*, npj Quantum Inf. **8** 98 (2022)]

Monogamy vs. transitivity





Given nonlocal $\vec{P}_{\mathrm{AB}}$:

- $\forall$ compatible $\vec{P}_{\mathrm{ABC}}$
- $\vec{P}_{\mathrm{BC}}, \vec{P}_{\mathrm{AC}}$ are local

2-input, 2-output correlation

[B. Toner *et al.*, arXiv:quant-ph/0611001 (2006)]

Given nonlocal $\vec{P}_{\mathrm{AB}}, \vec{P}_{\mathrm{BC}}$:

- $\forall$ compatible $\vec{P}_{\mathrm{ABC}}$
- $\vec{P}_{\mathrm{AC}}$ is nonlocal

Post-quantum no-signaling correlation

[S. Coretti *et al.*, PRL **107**,100402 (2011)]

Nonlocality transitivity

(Type of transitivity)

Nonlocality transitivity of correlations:

Given $\vec{P}_{\mathrm{AB}}, \vec{P}_{\mathrm{BC}} \notin \mathcal{L} \Rightarrow \vec{P}_{\mathrm{AC}} \notin \mathcal{L}$

(Realization)

[S. Coretti *et al.*, PRL **107**,100402 (2011)]:

$\exists\ \vec{P}_{\mathrm{ABC}} \notin \mathcal{Q}$

(not realizable in the quantum realm)

Nonlocality transitivity of quantum correlations:

Given $\vec{P}_{\mathrm{AB}}, \vec{P}_{\mathrm{BC}} \notin \mathcal{L} \Rightarrow \vec{P}_{\mathrm{AC}} \notin \mathcal{L}$

(compatible with some $\vec{P}_{\mathrm{ABC}} \in \mathcal{Q}$)

$\exists$?

Challenge:

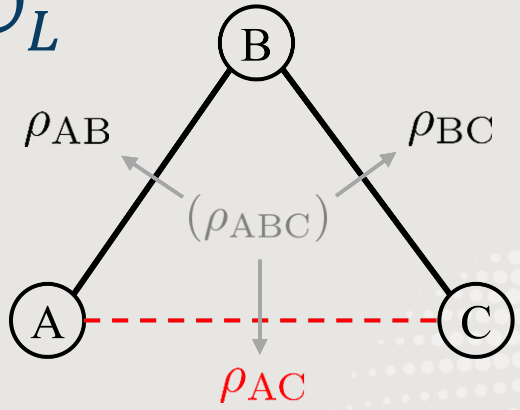
- The characterization of quantum $\vec{P}$ is difficult:

$$\text{Born's rule: } \mathrm{tr}\left(\rho_{\mathrm{AB}} M^A_{a|x} \otimes M^B_{b|y}\right) \underset{\text{hard!}}{\rightleftarrows} \vec{P} = \{P(a,b|x,y)\}$$

From correlations to states

A state is nonlocal $\rho_{AB} \notin \mathcal{D}_L$ if there are measurements that lead to a nonlocal correlation

Nonlocal transitivity for quantum states: $\rho_{\mathrm{AB}}, \rho_{\mathrm{BC}} \notin \mathcal{D}_L$ then $\forall \rho_{\mathrm{ABC}}: \rho_{\mathrm{AC}} \notin \mathcal{D}_L$



Prerequisite for nonlocality transitivity of quantum correlations

From correlations to states

- If the example of nonlocality transitivity of quantum correlations exists, i.e.,

$$\text{Given}\quad \begin{matrix} \vec{P}_{\mathrm{AB}} = \left\{\mathrm{tr}\left(\boxed{\rho_{\mathrm{AB}}} M^A_{a|x} \otimes M^B_{b|y}\right)\right\} \notin \mathcal{L} \\ \vec{P}_{\mathrm{BC}} = \left\{\mathrm{tr}\left(\boxed{\rho_{\mathrm{BC}}} M^B_{b|y} \otimes M^C_{c|z}\right)\right\} \notin \mathcal{L} \end{matrix} \quad \Rightarrow \vec{P}_{\mathrm{AC}} \notin \mathcal{L},$$

$$\{\vec{P}'_{\mathrm{AC}}\} \subseteq \{\vec{P}_{\mathrm{AC}}\}$$

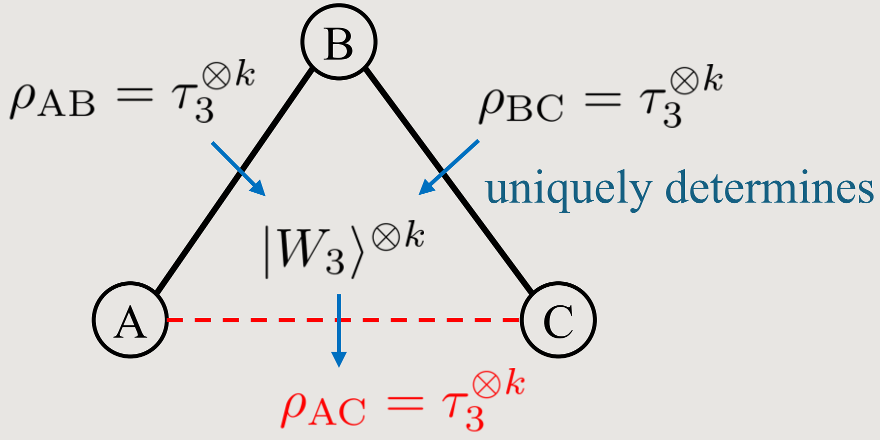
$$\vec{P}'_{\mathrm{AC}} = \left\{\mathrm{tr}\left(\rho_{\mathrm{AC}} M^A_{a|x} \otimes M^C_{c|z}\right)\right\} \notin \mathcal{L}$$

consider all compatible ρ_{AC}

$$(\rho_{\mathrm{AB}}, \rho_{\mathrm{BC}} \notin \mathcal{D}_{\mathcal{L}} \Rightarrow \rho_{\mathrm{AC}} \notin \mathcal{D}_{\mathcal{L}})$$

$\Longrightarrow$ an example of nonlocality transitivity of quantum states!

Construction from W-states



- $\tau_3 := \frac{2}{3}|\Psi^+\rangle\langle\Psi^+| + \frac{1}{3}|00\rangle\langle 00|,\ |\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$
- $|W_3\rangle = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle)$
- $\rho_{AB} = \rho_{BC} = \tau_3^{\otimes k}$ implies $\rho_{AC} = \tau_3^{\otimes k}$
- $\tau_3^{\otimes k}$ is nonlocal when $k \geq 29$ ($\tau_3^{\otimes 29} \notin \mathcal{D}_\mathcal{L}$) (witnessed using the Khot-Vishnoi game)

[C. Palazuelos, Phys. Rev. Lett. **109**, 190401 (2012)]
[D. Cavalcanti *et al.*, Phys. Rev. A **87**, 042104 (2013)]

Nonlocality transitivity of quantum states:

Given $\rho_{AB}, \rho_{BC} \notin \mathcal{D}_\mathcal{L} \Rightarrow \rho_{AC} \notin \mathcal{D}_\mathcal{L}$

$\rho_{AB} = \rho_{BC} = \tau_3^{\otimes 29}$

Construction from Haar-random states

- Given a Haar-random pure global state with local dimensions d:

$$|\Psi\rangle_{\mathrm{ABC}} \rightarrow \begin{matrix} \rho_{\mathrm{AB}} = \mathrm{tr}_{\mathrm{C}}\left(|\Psi\rangle\langle\Psi|\right) \\ \rho_{\mathrm{BC}} = \mathrm{tr}_{\mathrm{A}}\left(|\Psi\rangle\langle\Psi|\right) \end{matrix} \Rightarrow \rho_{AC} = \mathrm{tr}_{\mathrm{B}}\left(|\Psi\rangle\langle\Psi|\right) \text{ (uniqueness)}$$

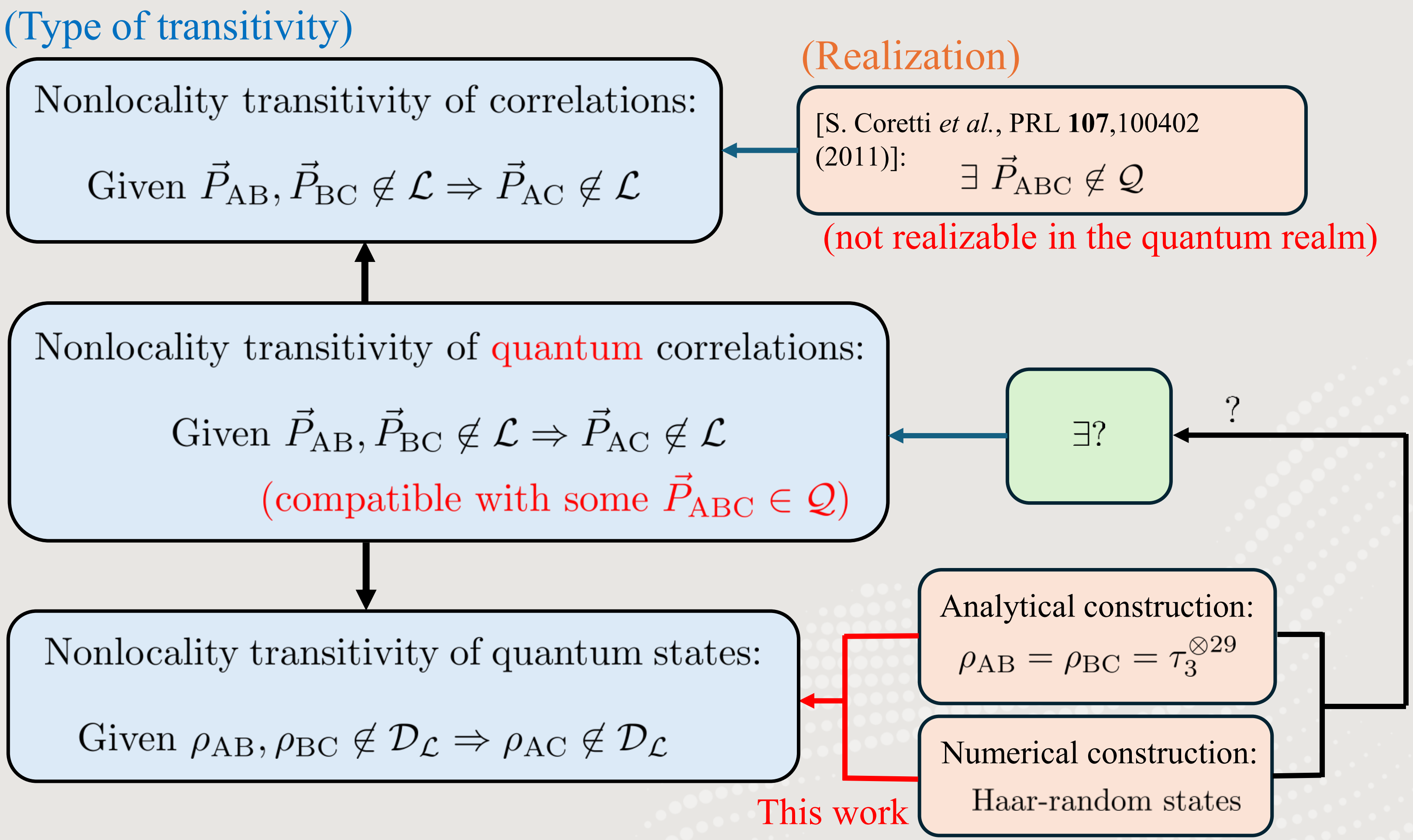
[M.-E. Liu *et al.*, Quantum Sci. Technol. **11** 015036 (2026)]

- Consider M dichotomic measurements per party:

Number of marginals found to be nonlocal

d	M	Inequality	Samples	None (%)	One (%)	Two (%)	All (%)
2	2	I_{CHSH}	100000	10.73	89.27	0	0
2	3	I_{3322}	100000	10.51	88.97	0.51	0
3	2	I_{CHSH}	100000	12.81	41.42	34.46	11.31
3	3	I_{3322}	100000	12.74	41.37	34.48	11.41
4	2	I_{CHSH}	10000	73.52	22.95	3.30	0.23
4	3	I_{3322}	10000	73.54	22.94	3.29	0.23
5	2	I_{CHSH}	1000	99.8	0.2	0	0
5	3	I_{3322}	1000	99.8	0.2	0	0

Nonlocality transitivity



Obstacles for correlations

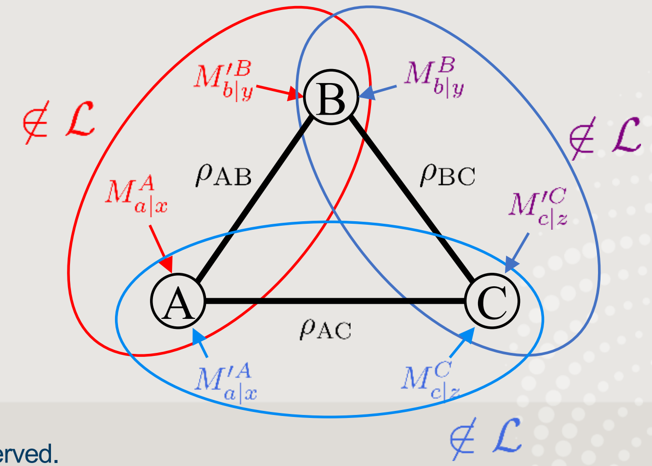
Measurement inconsistency

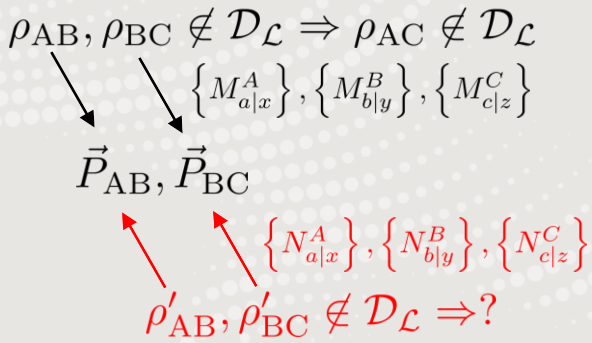
- Need same POVM to be nonlocal with other parties

Non-unique quantum realizations

- Different state and measurements give same correlation

More general no-signaling compatibility



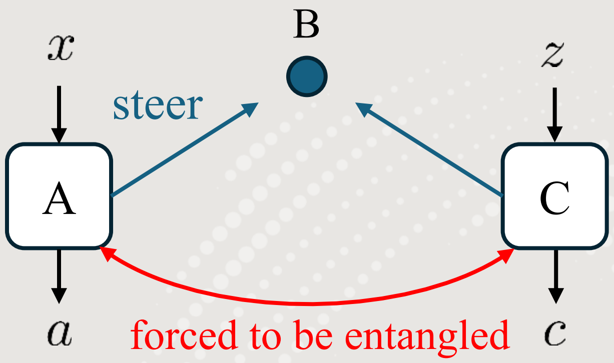


Steering transitivity

Marginals $\mathcal{W}_3$ of a W-state exhibit steering transitivity

- Let $H_d =$ be harmonic number
- Fully entangled fraction (FEF) $F(\rho)$ of ρ
- Let $F_d^\star = \frac{d+1}{d^2} H_d - \frac{1}{d}$
- Any 2-qudit state with FEF $F_d^\star$ is steerable
- For $d = 2$: we have $F(\mathcal{W}_3) = \frac{2}{3} > \frac{5}{8} = F_2^\star$

[M. T. Quintino *et al.*, Phys. Rev. A **94**, 062123 (2016)]



Partially device-independent: A and C can each steer B; measurement data suggest that A and C share entanglement

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(Type of transitivity)

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(compatible with some $\vec{P}_{\mathrm{ABC}} \in \mathcal{Q}$)

$\exists$?

?

Nonlocality transitivity of quantum states:

Given $\rho_{\mathrm{AB}}, \rho_{\mathrm{BC}} \notin \mathcal{D}_{\mathcal{L}} \Rightarrow \rho_{\mathrm{AC}} \notin \mathcal{D}_{\mathcal{L}}$

Analytical construction:
$\rho_{\mathrm{AB}} = \rho_{\mathrm{BC}} = \tau_3^{\otimes 29}$

Numerical construction:
Haar-random states

This work

Steering transitivity of quantum states:

Given $\rho_{\mathrm{AB}}, \rho_{\mathrm{BC}}$ steerable $\Rightarrow \rho_{\mathrm{AC}}$ steerable

Analytical construction:
$\rho_{\mathrm{AB}} = \rho_{\mathrm{BC}} = \tau_3$

Summary

First example of quantum-realizable nonlocality transitivity

- Multiple copies of W-state
- Haar-random pure qutrit tripartite states

npj Quantum Inf. (2026)

Marginals of W-state exhibit steering transitivity

Open questions:

- Does nonlocality transitivity exist for quantum correlations?
- Are there two-qubit marginals that exhibit nonlocality transitivity?
- Can nonlocality transitivity be extended to larger systems?


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of the earth,
Please do not waste the toilet paper.