

Fundamental Quality Bound on Optical Quantum Communication

based on arXiv:2510.07121

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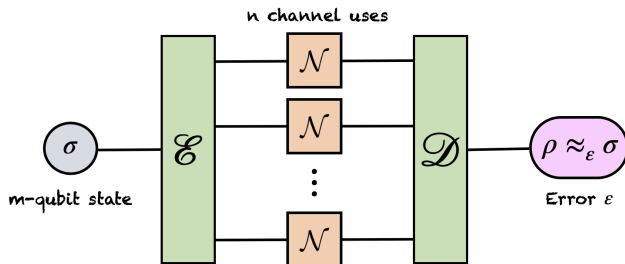
Optical Quantum Communication

Alice and Bob are connected by noisy optical channel $\mathcal{N}_{A \rightarrow B}$
 \leftrightarrow Mathematically: Infinite-dimensional bosonic CV-systems

Quantum Communication:

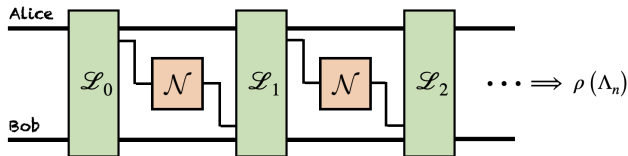
Using the channel n times, transmit m qubits, with error ε .

Moreover, any correlations with reference system should be preserved!



Two-Way LOCC Assistance

Assistance Model: Adaptive two-way LOCC operations $\Lambda_n \in \text{LOCC}_{\leftrightarrow}(\mathcal{N}^{\times n})$



\hookrightarrow Most-important practical assistance model

By quantum teleportation primitive: equivalent to entanglement distribution

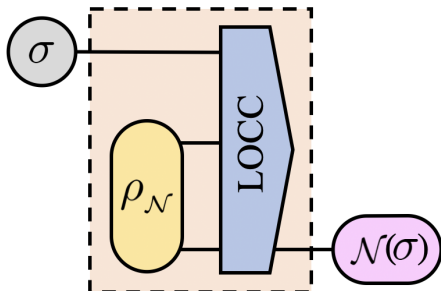
Entanglement Distribution:

Find two-way assisted adaptive protocol $\Lambda_n \in \text{LOCC}_{\leftrightarrow}(\mathcal{N}^{\times n})$ with output:

$$\rho(\Lambda_n) \approx_{\epsilon} \Phi^{\otimes m} \quad \text{with } |\Phi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Teleportation-Simulability

Most optical channels (e.g. thermal attenuator and amplifier, additive-Gaussian noise) are *teleportation-simulable* (with Choi state $\rho_{\mathcal{N}} := (\mathcal{N} \otimes \mathcal{I})(\Phi)$):



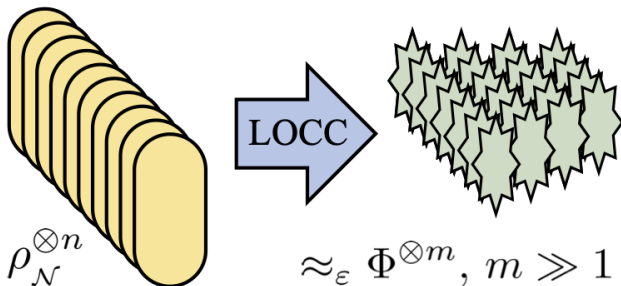
Adaptive-to-Block Reduction:¹

For any $\Lambda_n \in \text{LOCC}_{\leftrightarrow}(\mathcal{N}^{\times n})$: $\rho(\Lambda_n) = \bar{\Lambda}(\rho_{\mathcal{N}}^{\otimes n})$ with $\bar{\Lambda} \in \text{LOCC}_{\leftrightarrow}$

¹[Pirandola et al., Nat. Com. (2017)](see also [Bennett et al., Phys. Rev. (1996)])

The Quantity Picture

Traditionally: Maximize the number of transmitted qubits



Goal: Characterize the optimal achievable rate m/n

Quantum Capacity

Two-Way Assisted Quantum Capacity:

$$Q_{\leftrightarrow}(\mathcal{N}) := \lim_{\varepsilon \rightarrow 0} \sup \left\{ \lim_{n \rightarrow \infty} \frac{m(\Lambda_n)}{n} : \rho(\Lambda_n) \approx_{\varepsilon} \Phi^{\otimes m}, \Lambda_n \in \text{LOCC}_{\leftrightarrow}(\mathcal{N}^{\times n}) \right\}$$

For teleportation-simulable channels:

$$Q_{\leftrightarrow}(\mathcal{N}) = E_{d,\leftrightarrow}(\rho_{\mathcal{N}})$$

with the distillable entanglement of the Choi state $E_{d,\leftrightarrow}(\rho_{\mathcal{N}})$.

Best known upper-bound is the regularized REE:²

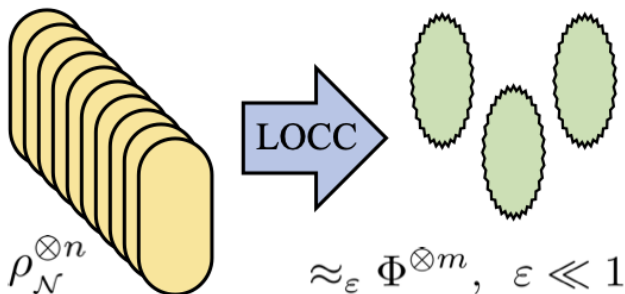
$$Q_{\leftrightarrow}(\mathcal{N}) \leq D^{\infty}(\rho \parallel \text{SEP}) := \lim_{n \rightarrow \infty} \frac{1}{n} \inf_{\sigma_n \in \text{SEP}} D(\rho^{\otimes n} \parallel \sigma_n) \leq D(\rho \parallel \text{SEP})$$

↪ Regularization is (1) generally necessary and (2) computationally very hard!

²[Pirandola et al., Nat. Com. (2017)] (see also [Bennett et al., Phys. Rev. (1996)])

A Shift in Perspective

Today: Following [Lami *et al.*, Nat. Phys. (2026)], maximise the quality of transmission.



Goal: Characterize the optimal exponent c with $\epsilon \sim 2^{-cn}$

Our guiding question

Can we give **quantitative bounds** on the **quality** of quantum communication and entanglement distribution over (very noisy) **optical channels** for which we do not have strong capacity characterisations?

First Main Finding: Yes!

Two-Way Assisted (Zero-Rate) Error Exponent:

$$Q_{\leftrightarrow, \text{err}}(\mathcal{N}) = \lim_{m \rightarrow \infty} \sup \left\{ \lim_{n \rightarrow \infty} -\frac{1}{n} \log \varepsilon(\Lambda_n) : \rho(\Lambda_n) \approx_\varepsilon \Phi^{\otimes m}, \Lambda_n \in \text{LOCC}_{\leftrightarrow}(\mathcal{N}^{\times n}) \right\}$$

For teleportation-simulable channel \mathcal{N} that acts on m bosonic modes:

$$Q_{\leftrightarrow, \text{err}}(\mathcal{N}) \leq \liminf_{r \rightarrow \infty} D(\text{SEP} \| \rho_{\mathcal{N}}(r)) =: E_{\text{RR}}(\mathcal{N})$$

with the *quasi*-Choi state

$$\rho_{\mathcal{N}}(r) = (\mathcal{N} \otimes \mathcal{I})(\Phi(r)^{\otimes m}),$$

where $\Phi(r)$ is two-mode squeezed vacuum state (with squeezing r).

↪ Crucially, regularisation is *superfluous* as $D(\text{SEP} \| \rho)$ is additive!

Proof Ingredients

For any achievable error exponent s :

- Use twirling of the output to restrict to protocols with isotropic output

$$\rho \left(\Lambda_n^{(m)} \right) = f_n \Phi^{\otimes m} + (1 - f_n) \tau_{2^m} \quad \text{with} \quad f_n \geq 1 - 2^{-ns}$$

for sufficiently large m and $n(m)$

- By teleportation-simulability $\rho \left(\Lambda_n^{(m)} \right) = \lim_{r \rightarrow \infty} \bar{\Lambda}_{n,r}^{(m)} \left(\rho_{\mathcal{N}}(r)^{\otimes n} \right)$:

$$D \left(\text{SEP} \left\| \rho \left(\Lambda_n^{(m)} \right) \right. \right) \leq \liminf_{r \rightarrow \infty} D \left(\text{SEP} \left\| \bar{\Lambda}_{n,r}^{(m)} \left(\rho_{\mathcal{N}}(r)^{\otimes n} \right) \right. \right) \quad (1)$$

$$\leq \liminf_{r \rightarrow \infty} D \left(\text{SEP} \left\| \rho_{\mathcal{N}}(r)^{\otimes n} \right. \right) \quad (2)$$

$$= n \liminf_{r \rightarrow \infty} D \left(\text{SEP} \left\| \rho_{\mathcal{N}}(r) \right. \right) \quad (3)$$

using lower semi-continuity (1), LOCC-monotonicity (2) and additivity (3).

- By isotropic symmetry of the output:

$$D \left(\text{SEP} \left\| \rho \left(\Lambda_n^{(m)} \right) \right. \right) = D_{\text{bin}}(2^{-m} \| f_n) \geq -1 + ns (1 - 2^{-m})$$

Second Main Finding

We have a **single-letter** upper bound on the quality of quantum communication, but is it actually **computable**?

Yes! For Gaussian states ρ_G with covariance matrix \mathbf{V}_ρ :

$$D(\text{SEP} \parallel \rho_G) = \min_{\mathbf{V}_\sigma, \gamma_A > 0} \frac{\text{tr}\{\mathbf{V}_\sigma(\mathbf{G}[\mathbf{V}_\rho] - \mathbf{G}[\mathbf{V}_\sigma])\}}{2 \ln(2)} + \log_2 \sqrt{\frac{\det(\mathbf{V}_\rho + i\Omega_{AB})}{\det(\mathbf{V}_\sigma + i\Omega_{AB})}}$$

s.t. $\mathbf{V}_\sigma \geq \gamma_A \oplus i\Omega_B$ and $\gamma_A \geq i\Omega_A$

with the symplectic form Ω and Gibbs matrix $\mathbf{G}[\mathbf{V}]$.

↔ Finite-dimensional convex program with two simple PSD constraints!

Proof Ingredients

Key Observation: For Gaussian states ρ_G : $D(\text{SEP} \parallel \rho_G) = D(\text{SEP}_G \parallel \rho_G)$

↪ It is strongly believed that this does not hold for $D(\rho_G \parallel \text{SEP})!$

Let σ_G be the *Gaussification* of σ (i.e. Gaussian state with same first and second moments):

$$\begin{aligned} D(\sigma \parallel \rho_G) &= -H(\sigma) - \text{tr}\{\sigma \log \rho_G\} \\ &\geq -H(\sigma_G) - \text{tr}\{\sigma_G \log \rho_G\} = D(\sigma_G \parallel \rho_G) \end{aligned}$$

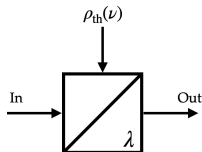
Technical Remark: We first show that w.l.o.g. one can restrict to separable states with finite second moments using a variational characterisation.

The convex program combines (1) separability criterion from [Lami *et al.*, New J. Phys. (2018)] with (2) the statistical moment characterization of the relative entropy from [Pirandola *et al.*, Nat. Com. (2017)].

Example: Thermal Attenuator (1/2)

Analytical solutions available for the most important one-mode Gaussian channels, i.e. thermal attenuator and amplifier, and additive-Gaussian noise

Thermal attenuator channel $\mathcal{N}_{\lambda, \nu}$ with transmissivity $\lambda \in [0, 1]$ and added thermal noise $\nu \in [0, \infty)$



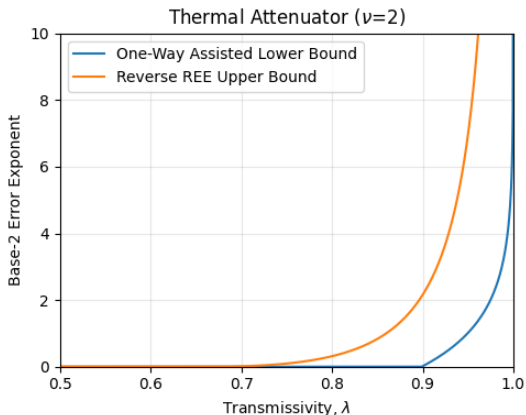
The (zero-rate) error exponent of the thermal attenuator is upper-bounded by

$$E_{\text{RR}}(\mathcal{N}_{\lambda, \nu}) = \log_2 [(1 + \nu)(1 - \lambda)] + \frac{\lambda}{1 - \lambda} \log_2 \left(\frac{\lambda}{\lambda_{\min}} \right) \text{ with } \lambda_{\min} := \frac{\nu}{\nu + 1}$$

for $\lambda \in [\lambda_{\min}, 1]$ and zero otherwise.

Example: Thermal Attenuator (2/2)

Achievability Question: Lifted *one-way assisted* random-coding-based bound from [Berta *et al.*, arXiv:2602.17430] to one-mode Gaussian channels



Bonus: Operational Interpretation

Exact operational interpretation in entanglement distillation with *non-entangling* operations:

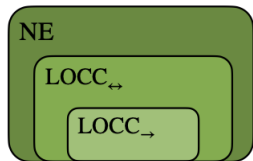
$$D(\text{SEP} \parallel \rho) = E_{d,\text{err}}^{\text{NE}}(\rho),$$

where $E_{d,\text{err}}^{\text{NE}}(\rho_{AB})$ is (zero-rate) error exponent of entanglement distillation .

↪ Extension of the finite-dimensional result from [Lami *et al.*, Nat. Phys. (2026)] to *general* (separable) infinite-dimensional quantum systems

Technical Remark: We prove the generalised quantum Sanov theorem from [Lami *et al.*, Nat. Phys. (2026)] in *general* (separable) infinite-dimensional quantum systems

The reverse relative entropy of entanglement is thus the first *operational* entanglement measure that is also *efficiently computable* for Gaussian states.



Conclusion

Takehome Message:

The quality perspective provides a promising alternative to study optical quantum communication, circumventing many roadblocks of the capacity framework.

Some future directions:

- Improvement of achievability result?
- Extensions to other (Gaussian) resource theories?

Based on [arXiv:2510.07121](https://arxiv.org/abs/2510.07121), please see for details the full paper:

