Clock precision is not limited by the 2$^{\text{nd}}$ law

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Read the paper :)

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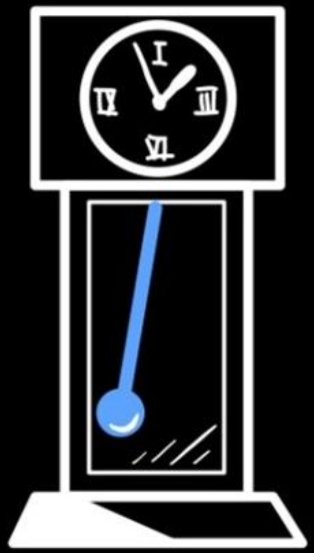
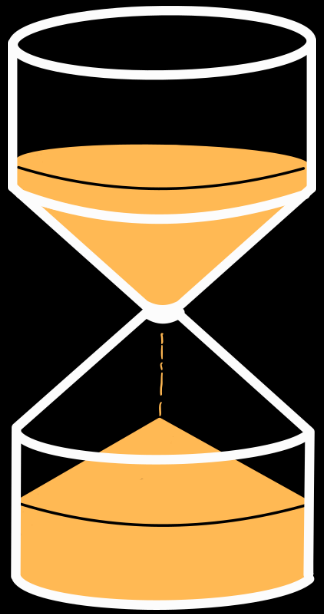
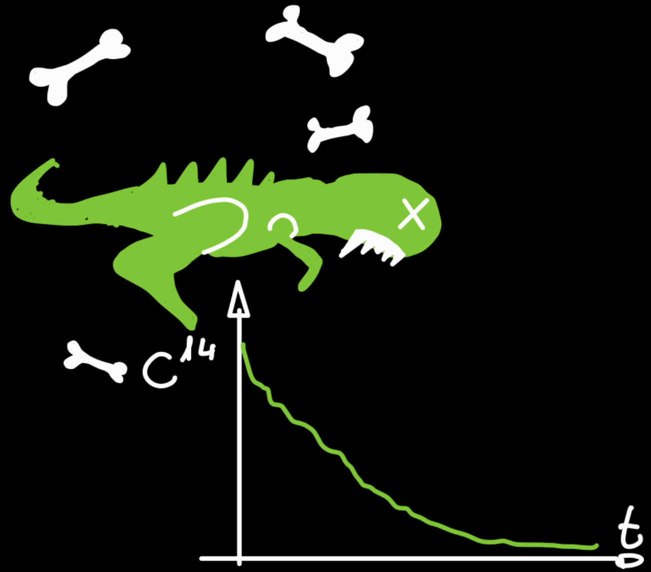
ASPECTS

What is a clock?



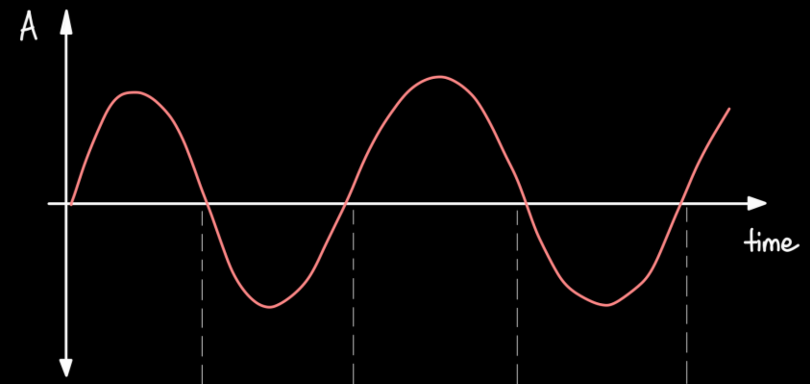




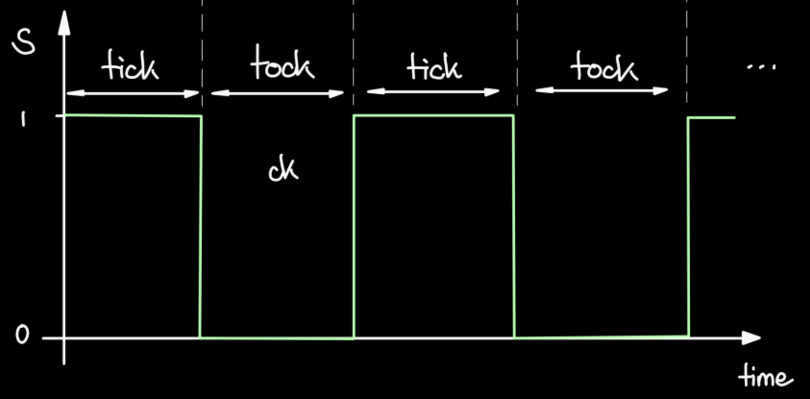
Outline

I. 'Ticking' clocks

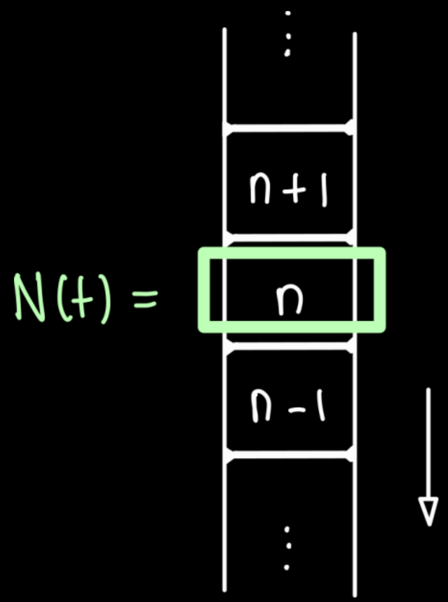
- Oscillatory process



- Discretization



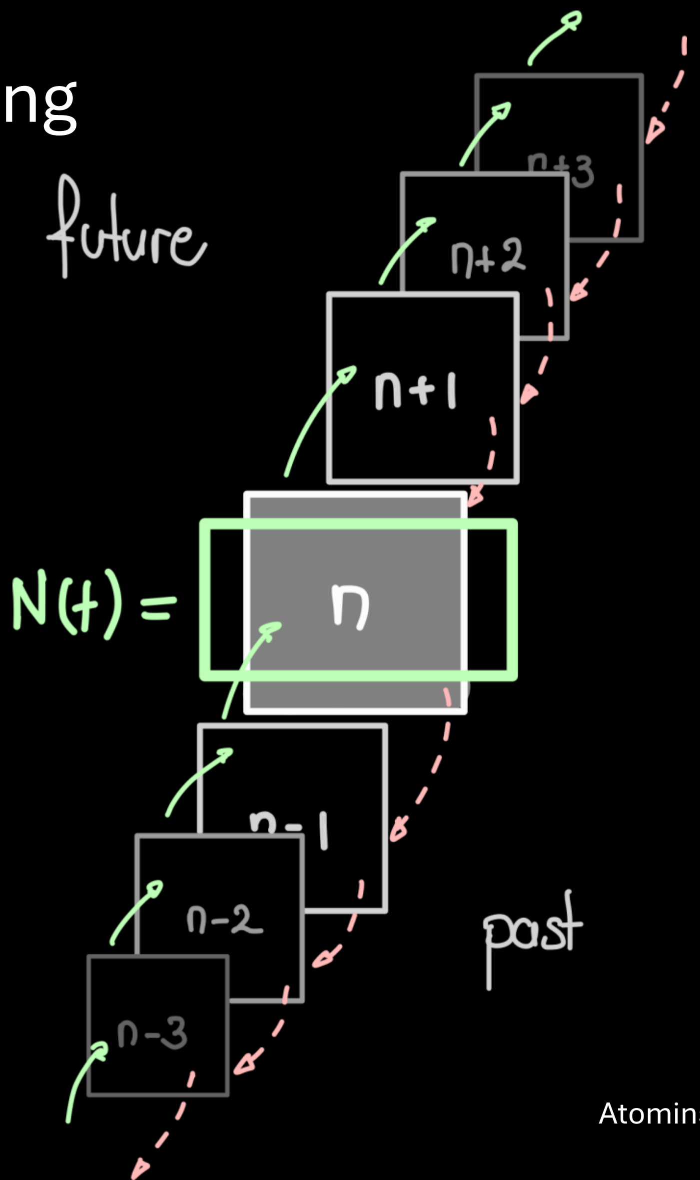
→ Counter



Ref: G. J. Milburn, *The thermodynamics of clocks*, Contemporary Physics **61**, 69 (2020)

I. Counting is irreversible

- Counting



- Crooks' fluctuation theorem

$$\frac{p(\text{backward})}{p(\text{forward})} = e^{-\Sigma_{\text{tick}}}$$

$\Downarrow$

entropy per tick Σ_{tick}

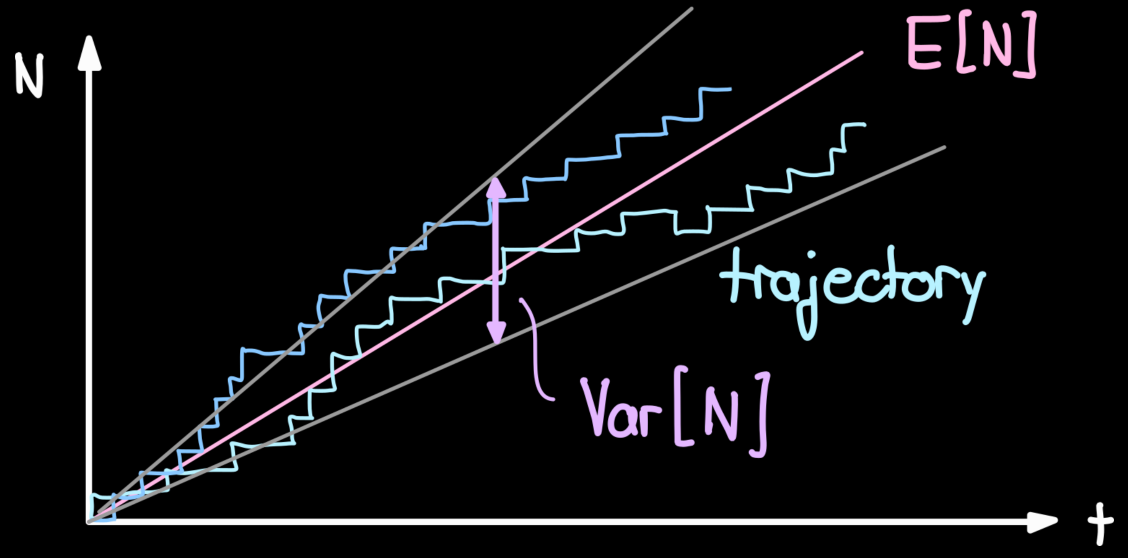
I. Figures of merit

- Number of ticks

$$\hat{N}(t) = \hat{N}_{\text{forward}}(t) - \hat{N}_{\text{backward}}(t)$$

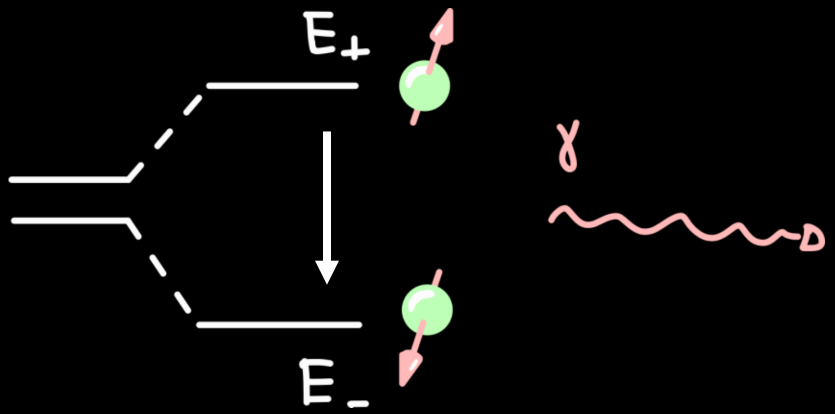
- Precision: Inverse Fano Factor

$$\mathcal{N} = \lim_{t \to \infty} \frac{E[N(t)]}{\mathrm{Var}[N(t)]}$$

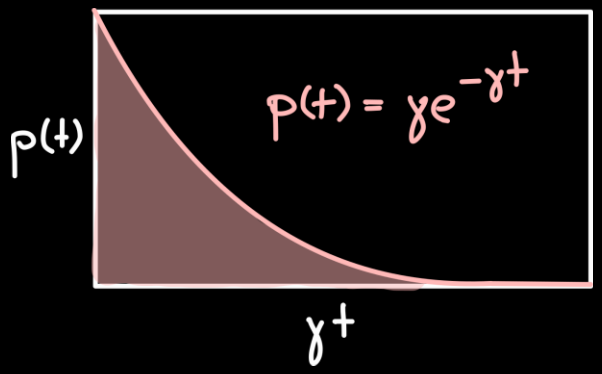


II. Minimal clock model

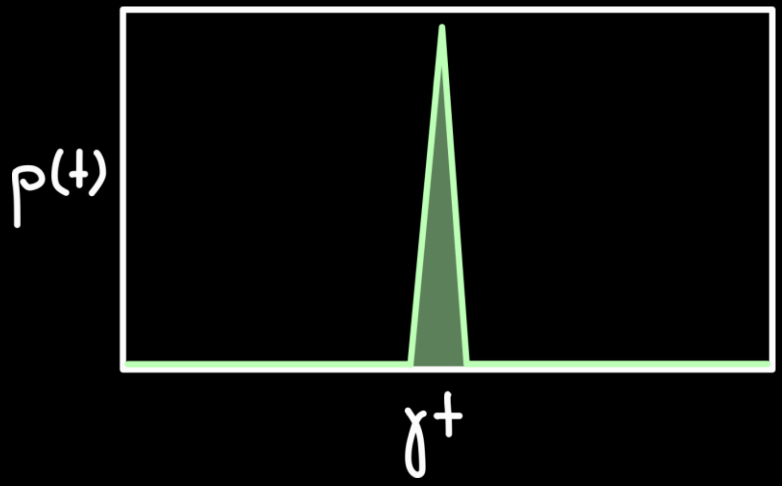
Decay Clock



Tick probability

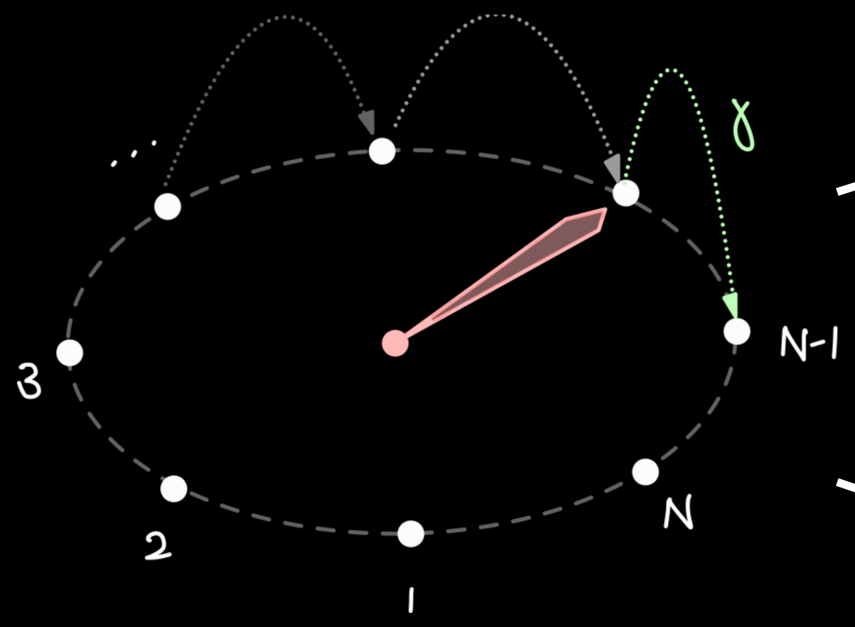


What we would want

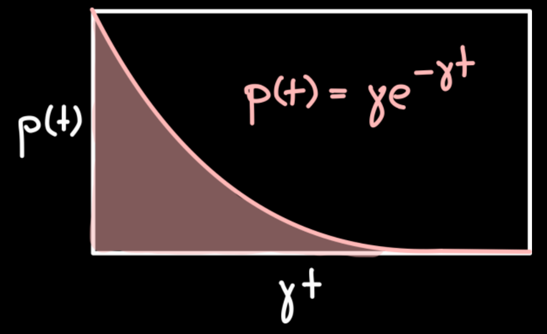


II. How to build a good clock

Markov chain clockwork



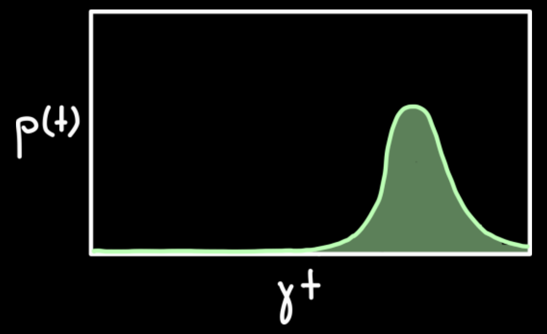
Count every click



$$\mathcal{N} \sim 1$$

$$\Sigma_{\text{tick}} \sim \mathcal{O}(1)$$

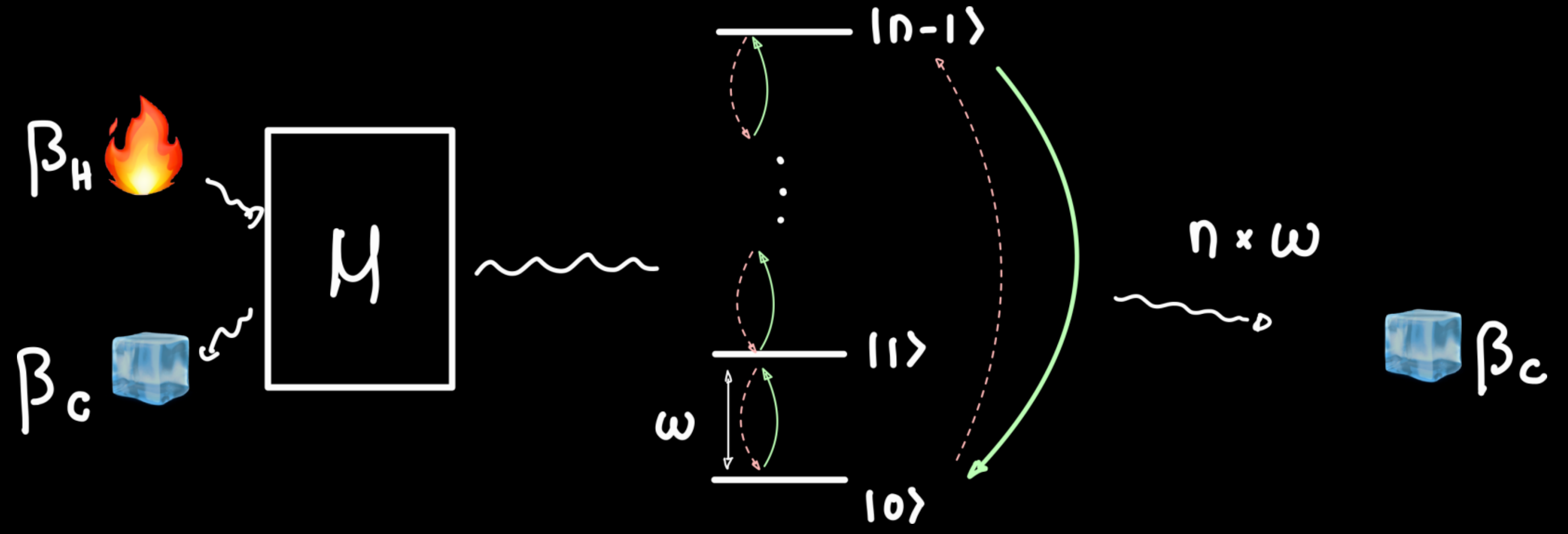
Count every cycle



$$\mathcal{N} \sim N$$

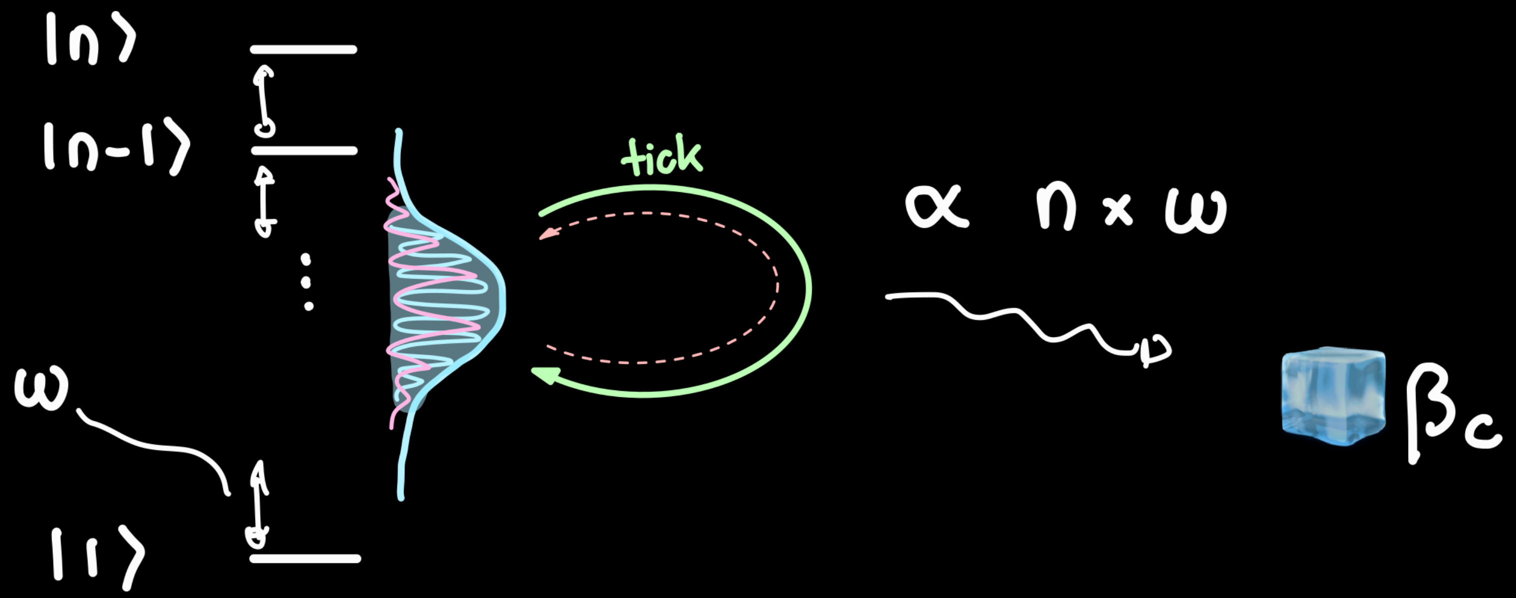
$$\Sigma_{\text{tick}} \sim \mathcal{O}(N)$$

II. A (semi)-classical example



Ref: P. Erker et al., Phys. Rev. X **7**, 031022 (2017), K. Erlang, Post Office Electrical Engineer's Journal **10**, 189 (1917)

II. A quantum clock



Ref: M. P. Woods et al., Annales Henri Poincaré **20**, 125 (2019), A. P. T. Dost and M. P. Woods, preprint arXiv:2303.10029 [quant ph] (2023).

II. Entropy as a resource

- Classical thermodynamic uncertainty relation

$$\mathcal{N}_c \sim n \quad \& \quad \Sigma_{tick} \sim n\beta\omega \quad \Rightarrow \quad \boxed{\mathcal{N}_c \leq \frac{\Sigma_{tick}}{2}}$$

- Conjectured quantum bound

$$\mathcal{N}_q \sim n^2 \qquad \Sigma_{tick} \sim n\beta\omega \quad \Rightarrow \quad \mathcal{N}_q \overset{?}{\lesssim} \Sigma_{tick}^2$$

TUR Ref: A. C. Barato and U. Seifert, Phys. Rev. Lett. **114**, 158101 (2015), and P. Pietzonka, et al., Phys. Rev. E **96**, 012101 (2017)
Q Ref: A. P. T. Dost and M. P. Woods, preprint arXiv:2303.10029 [quant ph] (2023)

II. Entropy as a resource

- Classical thermodynamic uncertainty relation

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- Conjectured quantum bound

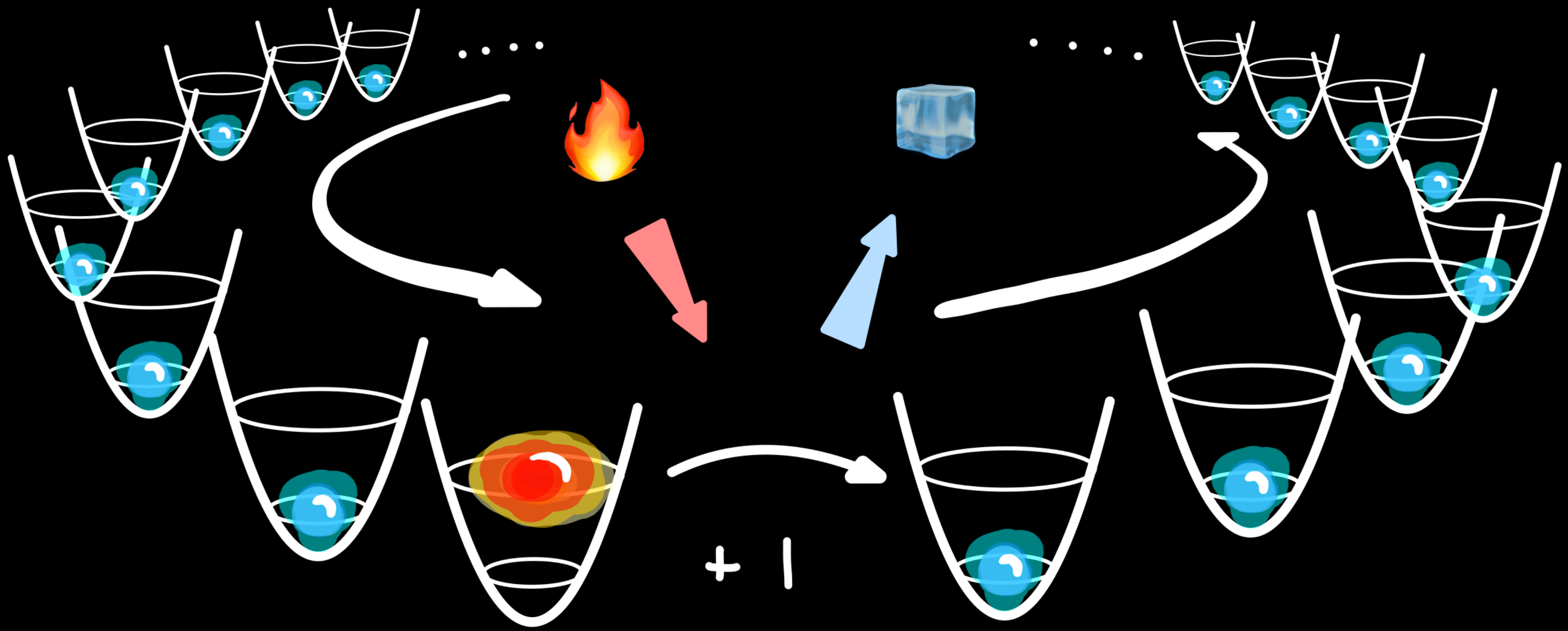
$$\mathcal{N}_q \sim n^2 \quad \quad \Sigma'_{tick} \sim n\beta\omega \quad \Rightarrow \quad \cancel{\mathcal{N}_q \overset{?}{\lesssim} \Sigma_{tick}^2}$$

here: YES

generally: NO

TUR Ref: A. C. Barato and U. Seifert, Phys. Rev. Lett. **114**, 158101 (2015), and P. Pietzonka, et al., Phys. Rev. E **96**, 012101 (2017)
Q Ref: A. P. T. Dost and M. P. Woods, preprint arXiv:2303.10029 [quant ph] (2023)

III. How can we do better?



III. Evolution generators

- Hamiltonian

$$H = \sum_{i=0}^{n-2} g_i \, \sigma_i^+ \sigma_{i+1}^- + h.c. \xrightarrow{\text{1-particle subspace}} H = \sum_{i=0}^{n-2} g_i \, |i+1\rangle\langle i| + h.c.$$

$n-2$: # sites

g_i: coupling

$$|i\rangle = |\underbrace{0, \dots, 0}_{i-1}, 1, \underbrace{0, \dots}_{n-i-1}\rangle$$

1: ith site

- Forward tick $\quad J = \sqrt{\Gamma}\, |0\rangle\langle n-1|$

- Backward tick $\quad \bar{J} = e^{-\Sigma_{tick}/2} J^\dagger$

III. Open quantum system's description

- Master equation evolution

$$\rho \in S(\mathcal{H}_1) \rightsquigarrow \dot{\rho} = -i[H, \rho] + \mathcal{D}_J[\rho] + \mathcal{D}_{\bar{J}}[\rho]$$

- Forward dissipator

$$\mathcal{D}_J[\rho] = J\rho J^\dagger - \frac{1}{2}\{J^\dagger J, \rho\}$$

- Backward dissipator

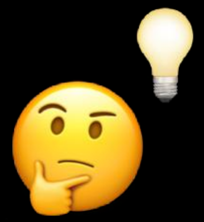
$$\mathcal{D}_{\bar{J}}[\rho] = e^{-\Sigma_{tick}}\left(J^\dagger \rho J - \frac{1}{2}\{J J^\dagger, \rho\}\right)$$

III. What now?

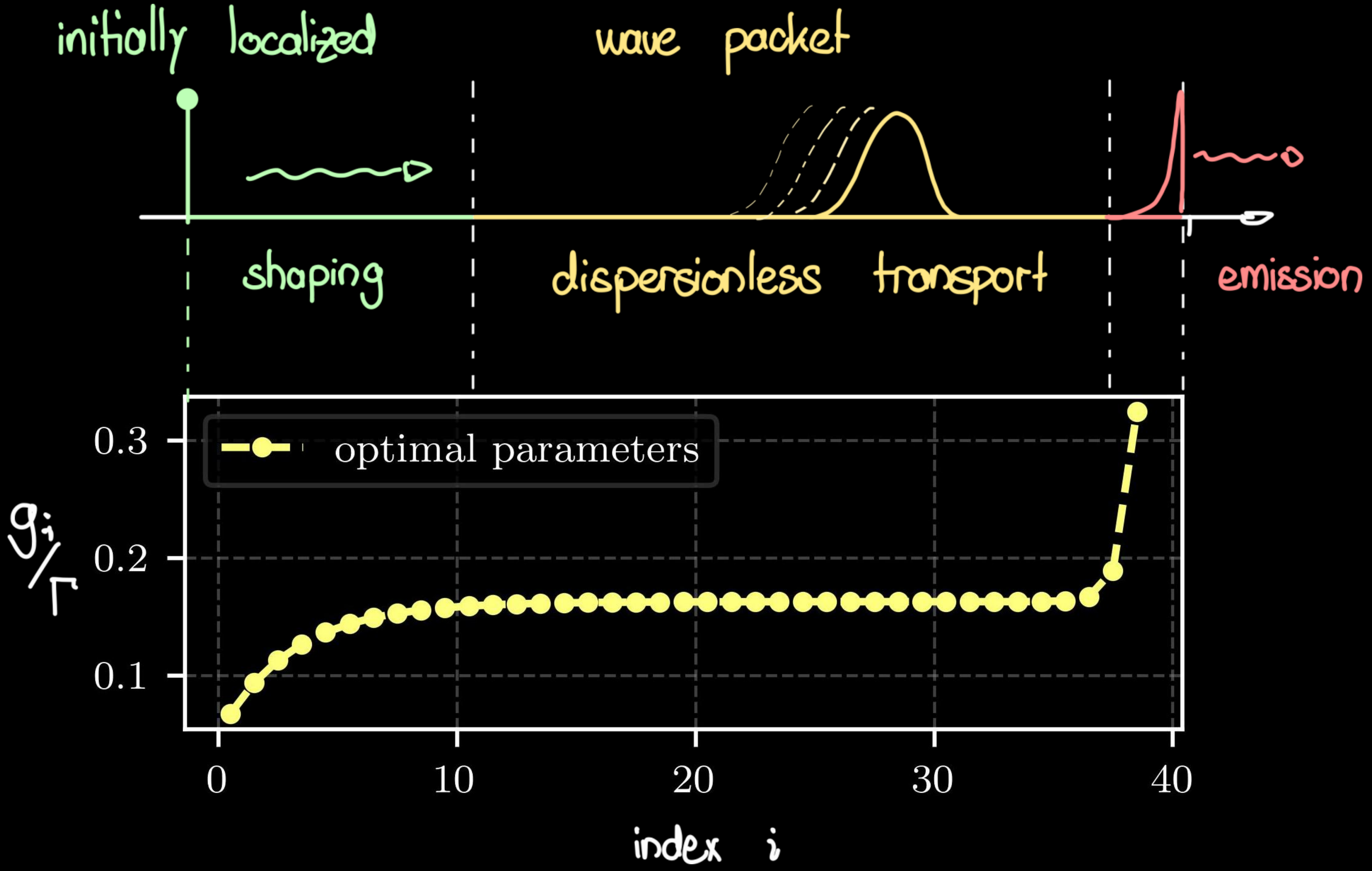
1. Brute force: Find optimal choice of couplings?

$$(g_i)_i = \underset{(g_i)_i}{\operatorname{argmax}} \{ \mathcal{N} \}$$

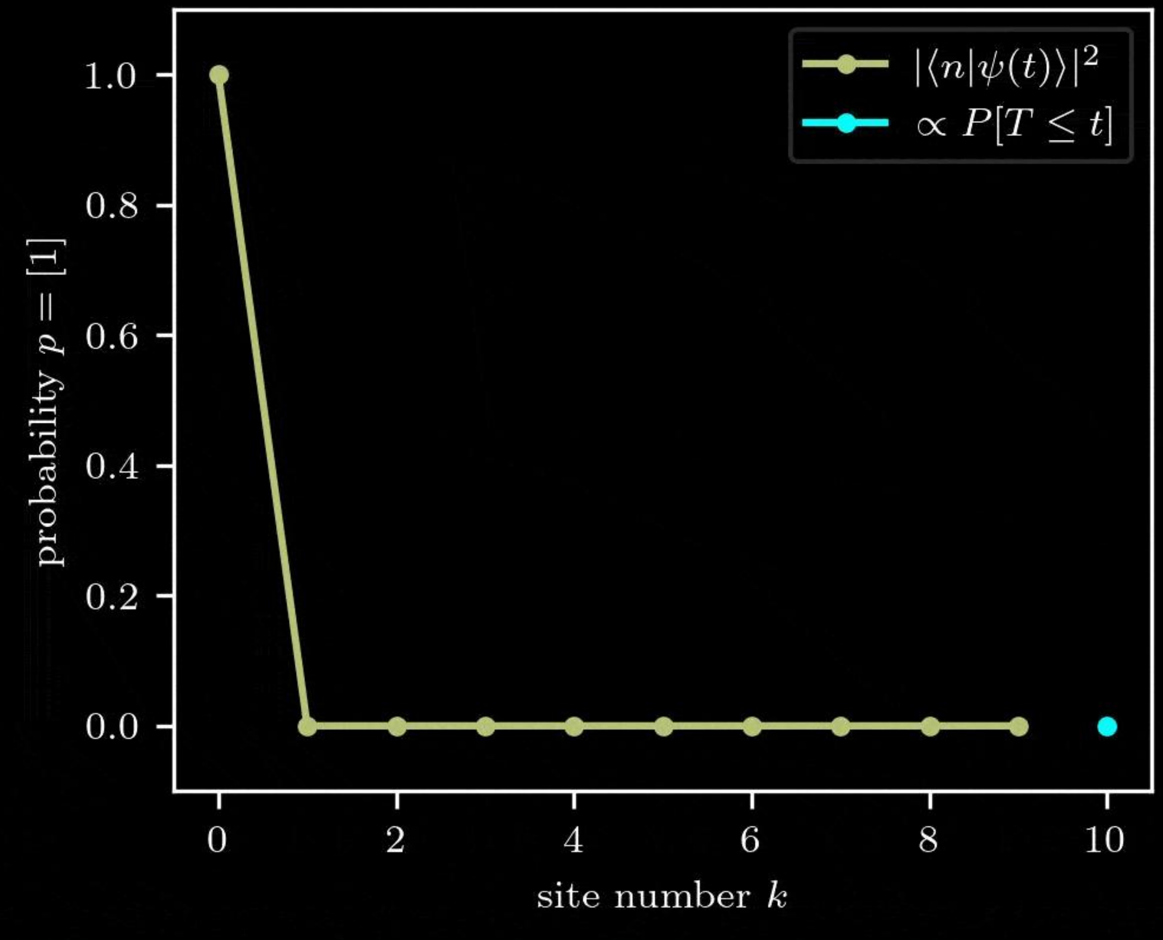
2. Understand why optimal



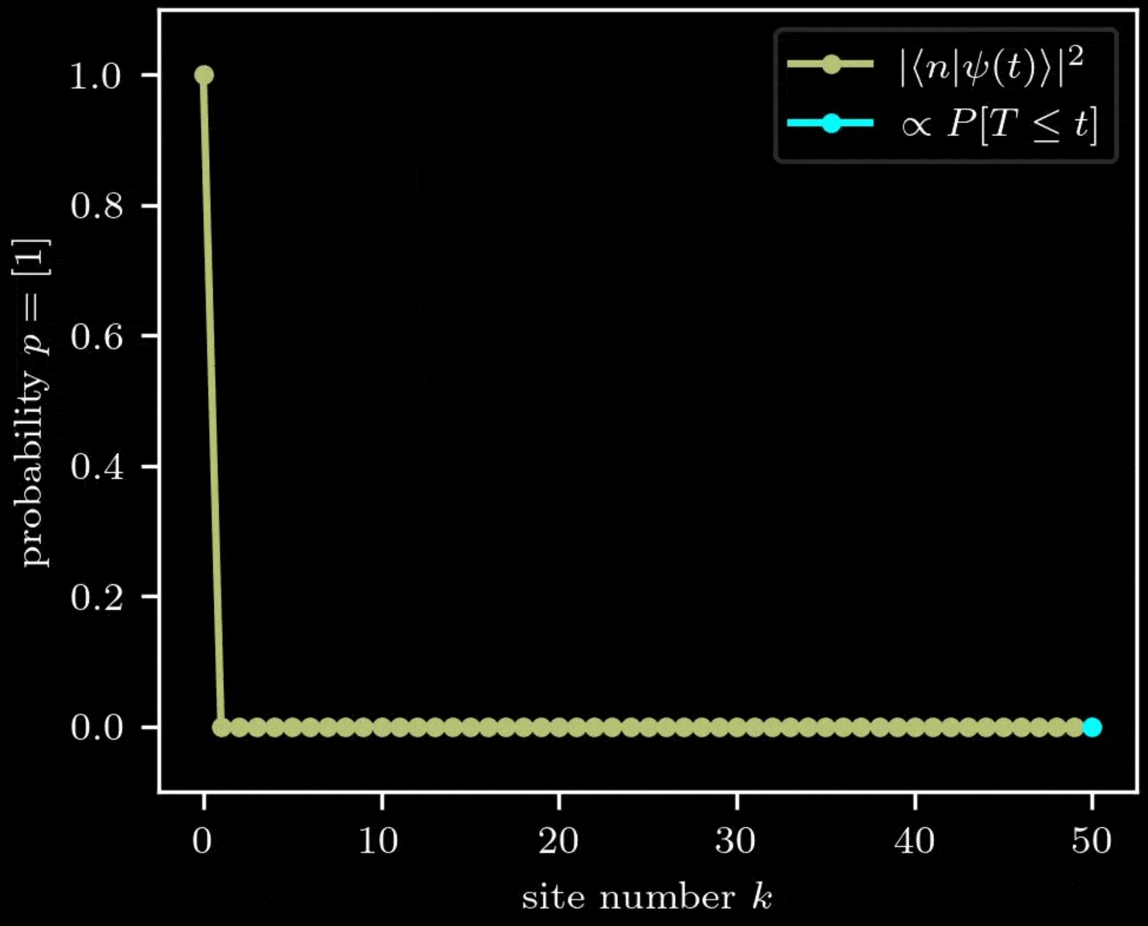
III. Optimized couplings



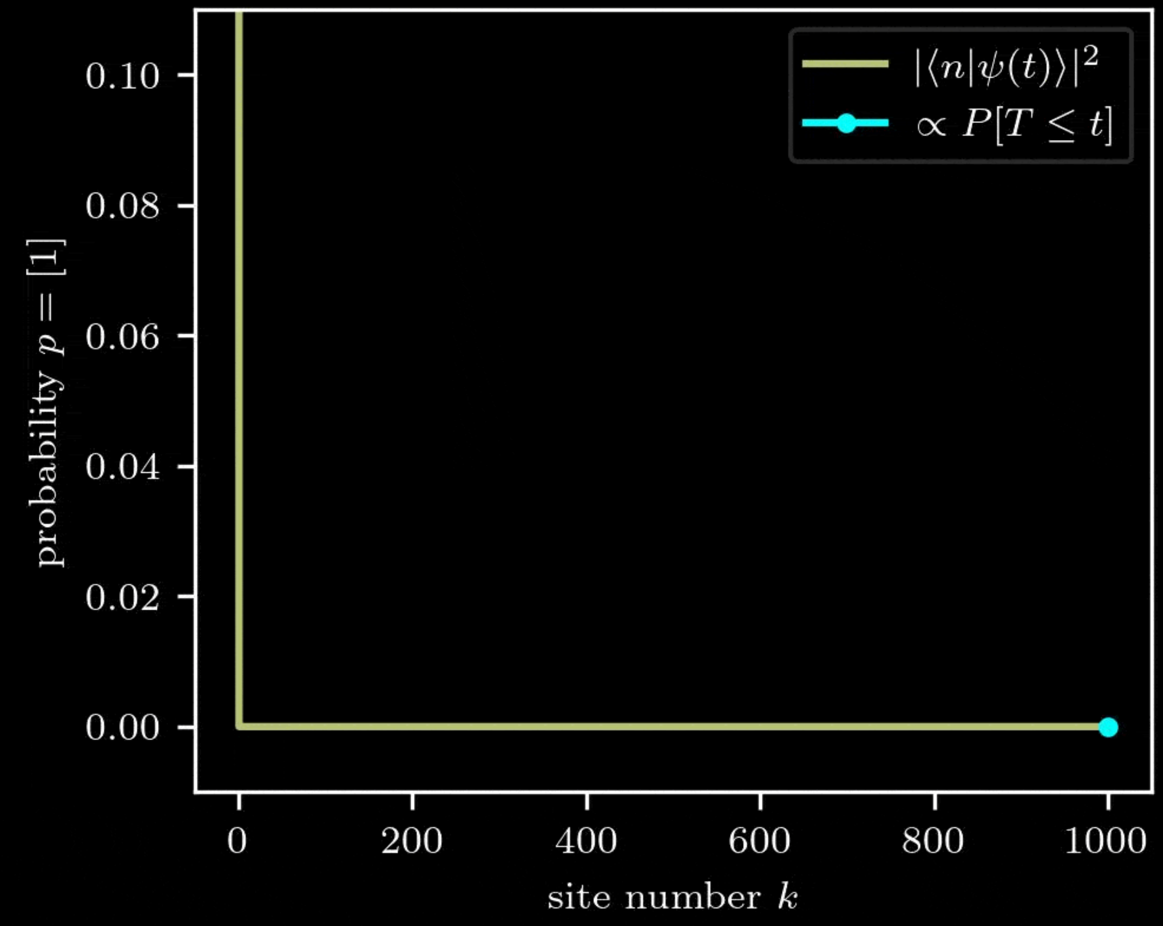
III. Dispersion-free evolution, n=10



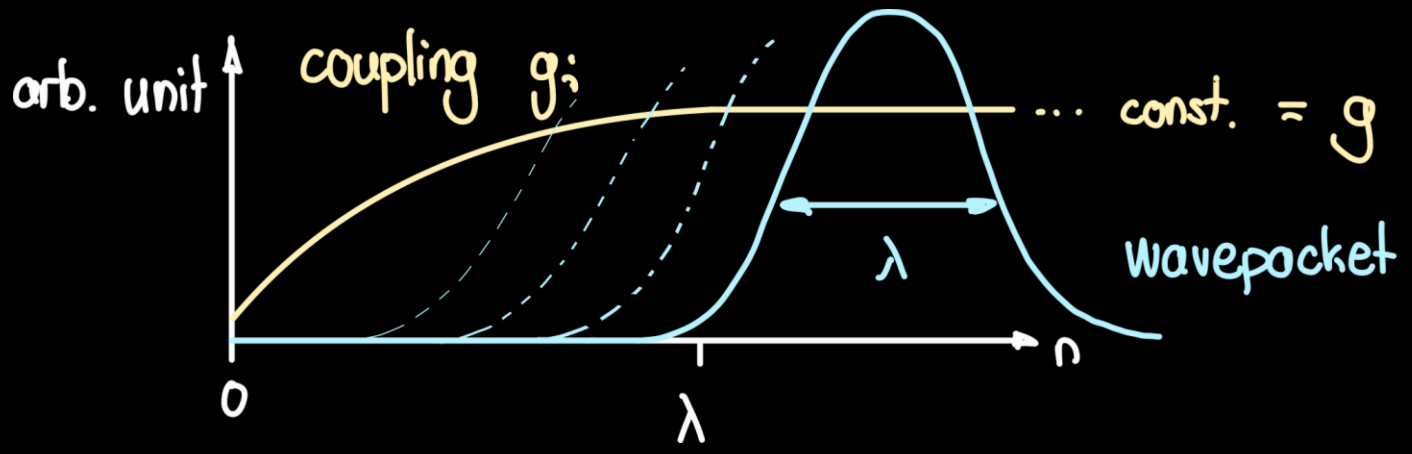
III. Dispersion-free evolution, n=50



III. Dispersion-free evolution, n=1000



III. Preparation ramp – hydrodynamical limit



- Wave packet evolution

mean position ~ $2gt$, width ~ λ

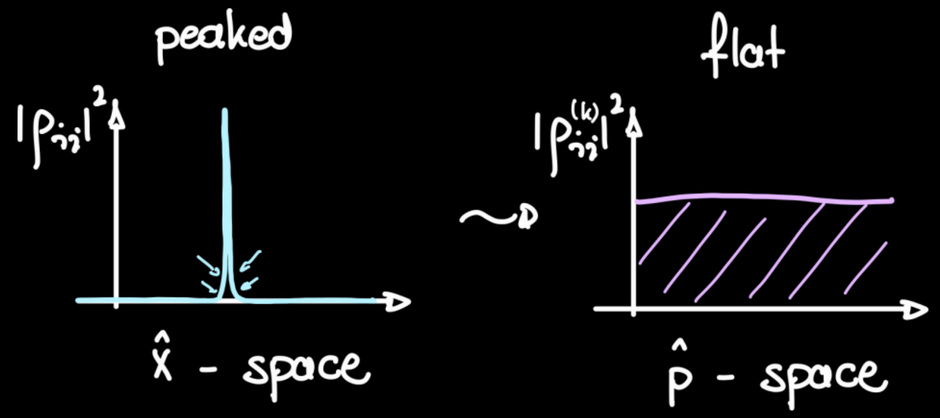
- Small λ = Good clock?

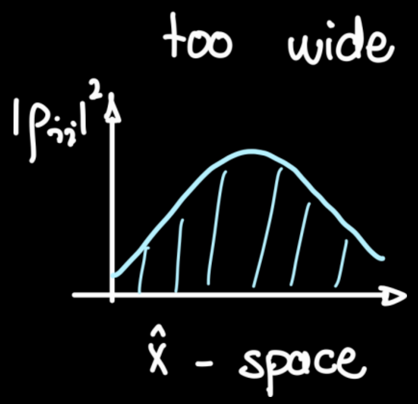
III. Bulk transport – hydrodynamical limit

Two competing effects

$\lambda \to 0$

$\lambda \to \infty$





dispersion

wavepacket width

Optimal choice → $\boxed{\lambda \sim n^{1/3}}$

III. Asymptotic scaling

- Asymptotic precision

now: $E[T] \sim n/g, \quad \mathrm{Var}[T] \sim n^{2/3}/g \quad \Rightarrow \quad \boxed{\mathcal{N} \sim n^{1+1/3}}$

still OK

error negligible

- Logarithmic entropy sufficient

$$\Sigma_{\text{tich}} \sim \alpha \log(n) \quad \Rightarrow \quad \| \mathcal{D}_{\bar{j}} \| \sim e^{-\Sigma_{\text{tich}}} \sim n^{-\alpha}$$

III. Result summary

- Precision

$$\mathcal{N} \sim n^{1+1/3}$$

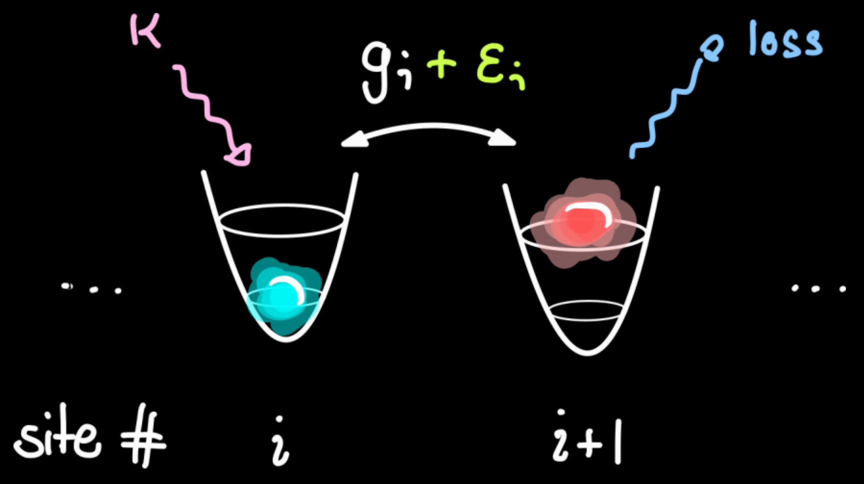
- Entropy

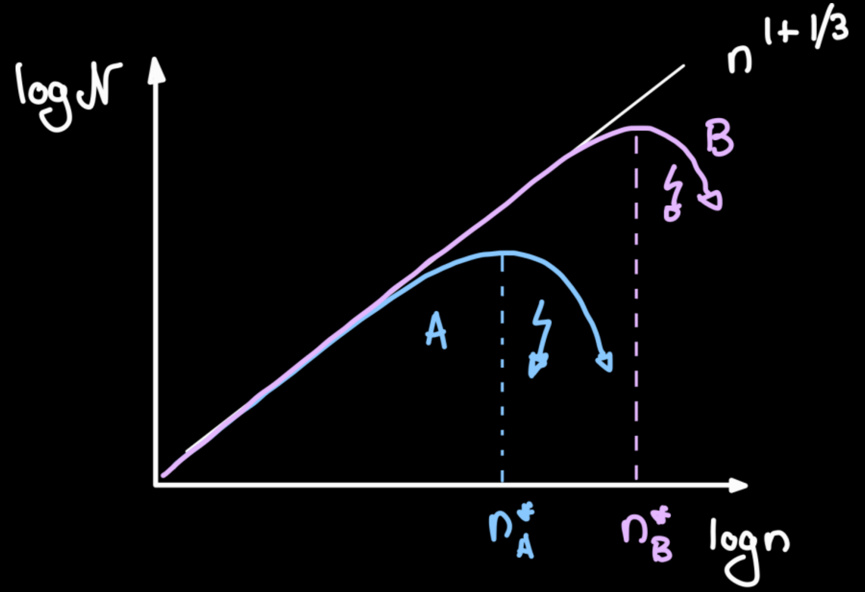
$$\Sigma_{tick} \sim \log(n)$$

$$\boxed{\mathcal{N} = e^{\Omega(\Sigma_{tick})}}$$

IV. Discussion: imperfections

Perturbed couplings & loss → finite scaling





error A > error B

$n_A^* < n_B^*$

IV. Main take-aways

Read the paper :)

- Clocks are dissipative systems

- Entropy production as a resource for precision

- Circumvent classical constraints with coherent evolution

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