Quantum Resources 2026

Tokyo, Japan
March 2026

imagine the impossible

KIAS KOREA INSTITUTE FOR ADVANCED STUDY

Limitations of stabilizer operations for discriminating stabilizer states

For more information: arXiv:2509.25790
(will be updated soon)

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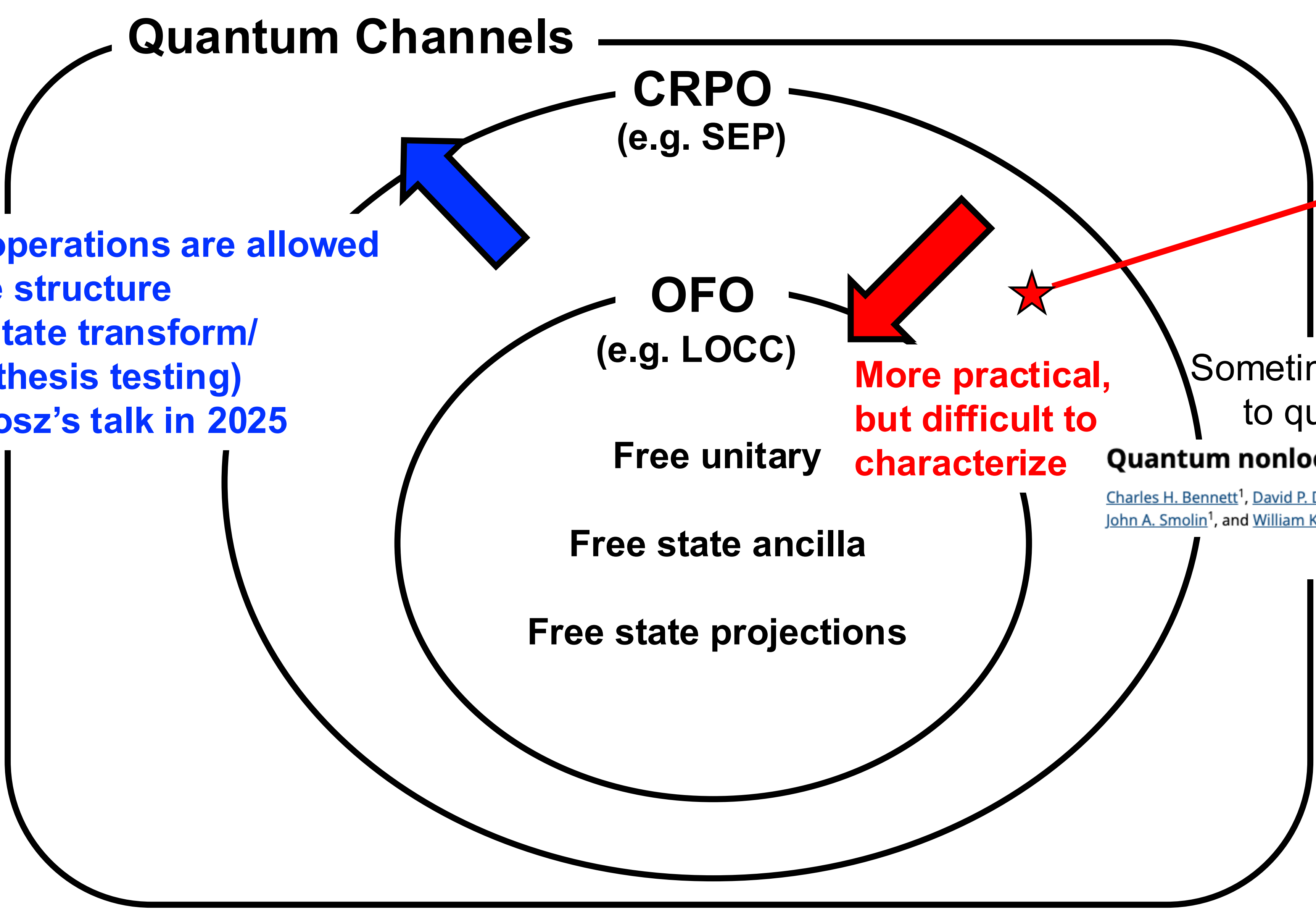
Motivation: Axiomatic vs. Operational free operations

- **Quantum Resource Theory:** $\mathcal{R} = (\mathcal{F}, \mathcal{O})$
  - $\mathcal{F}$: A set of (resource-) free states. (e.g. separable states, incoherent states)
  - $\mathcal{O}$: (Resource-)free operations. $\mathcal{O}(\mathcal{F}) \subset \mathcal{F}$

- **How do we characterize free operations?**
  - **Axiomatic approach**: All possible quantum channels that preserves resource
    (c.f. **C**ompletely **R**esource **P**reserving **O**perations: $(\Phi \otimes \mathrm{id.})(\rho) \in \mathcal{F}$ for any $\rho \in \mathcal{F}$)
    (e.g., **SEP**arable operations)
  - **Operational approach (OFO)**: Operations that we can actually implement via
    Free state prep., free unitary, and free measurements (proj. onto free states)
    (e.g., **L**ocal **O**perations and **C**lassical **C**ommunication)

Motivation: Axiomatic vs. Operational free operations



More operations are allowed
➲ Nice structure
(e.g., state transform/ hypothesis testing)
⋆Bartosz's talk in 2025

More practical, but difficult to characterize

When OFO is strictly less powerful than CROP
(e.g., LOCC ⊊ SEP)

Sometimes, this gap is closely related to quantum state discrimination

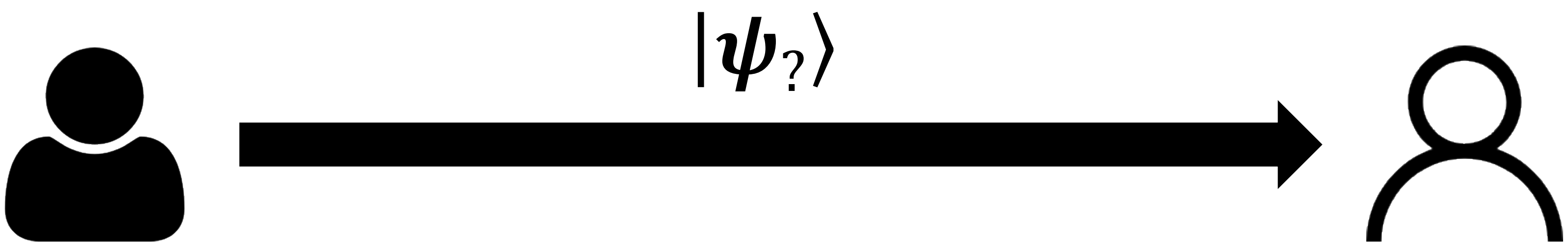
Quantum nonlocality without entanglement

Charles H. Bennett[1], David P. DiVincenzo[1], Christopher A. Fuchs[2], Tal Mor[3], Eric Rains[4], Peter W. Shor[4], John A. Smolin[1], and William K. Wootters[5]

[Phys. Rev. A 59, 1070 (1999)]

"Quantum Data Hiding"
⋆Francesco's talk on Wed.

Quantum state discrimination



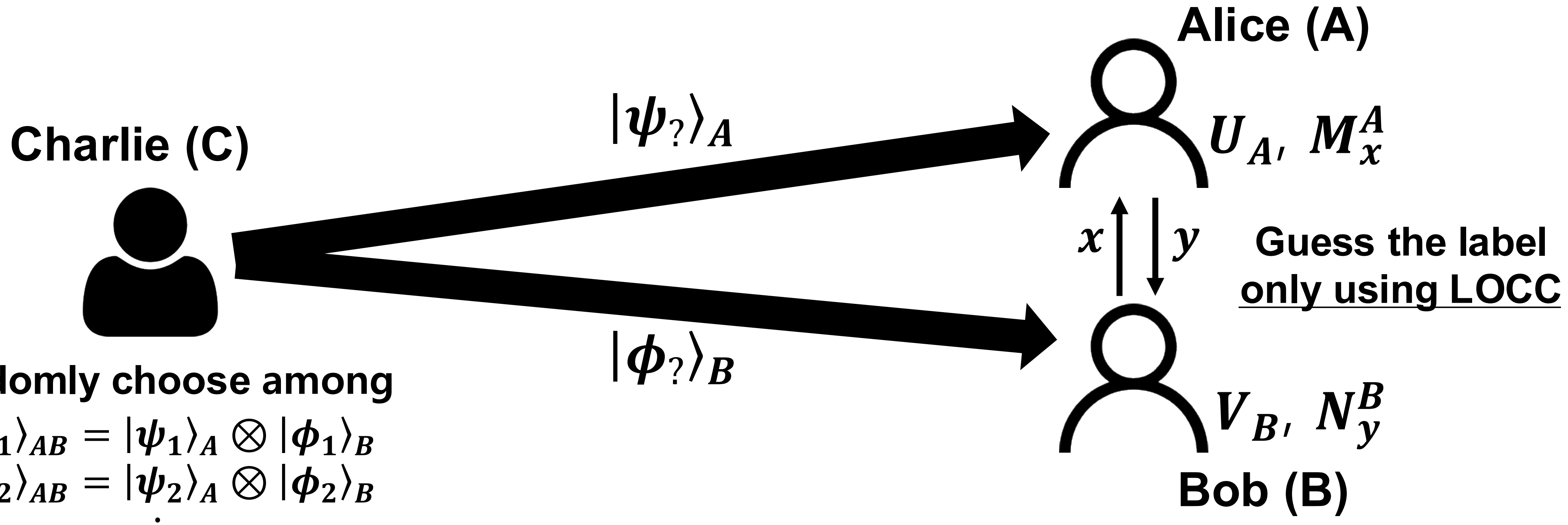
“Randomly choose”
among $|\psi_1\rangle, |\psi_2\rangle, \cdots, |\psi_k\rangle$

(State preparation)

“Guess whether the state is”
$? = 1, 2, \cdots, \text{or}, k$

(via Measurement)

Nonlocality without entanglement



Randomly choose among

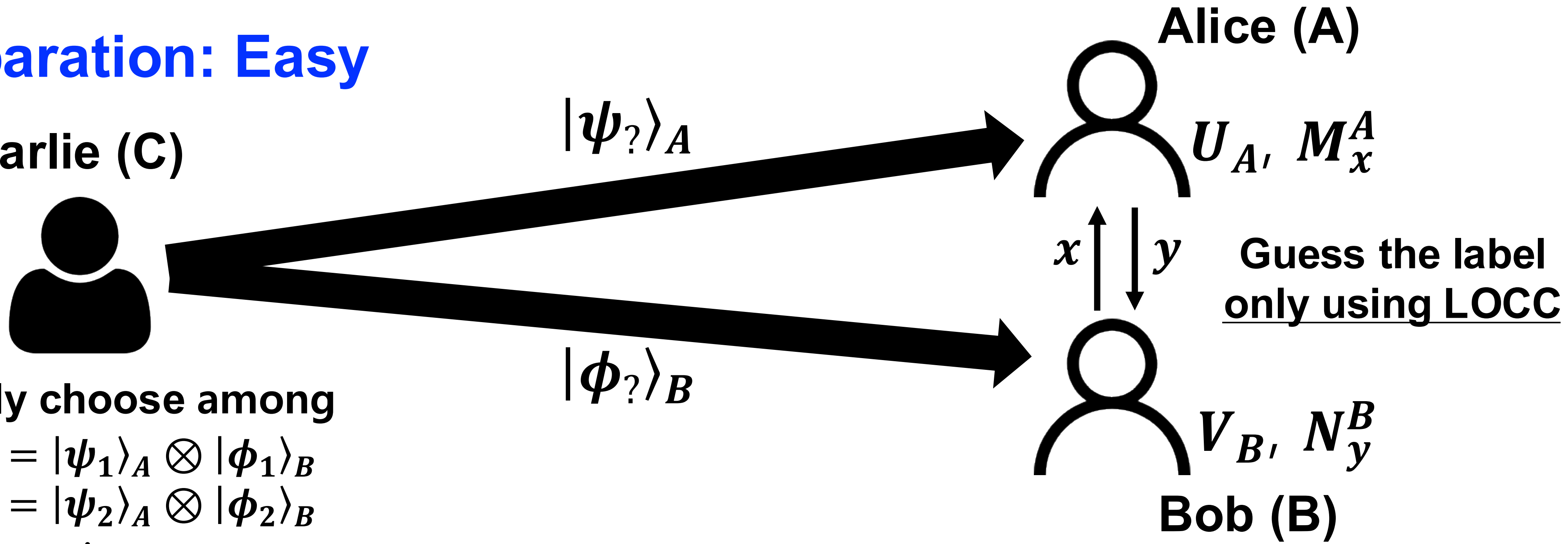
$$|\Psi_1\rangle_{AB} = |\psi_1\rangle_A \otimes |\phi_1\rangle_B$$
$$|\Psi_2\rangle_{AB} = |\psi_2\rangle_A \otimes |\phi_2\rangle_B$$
$$\vdots$$
$$|\Psi_k\rangle_{AB} = |\psi_k\rangle_A \otimes |\phi_k\rangle_B$$

* Ψ_j are orthogonal, but ψ_j, ϕ_j might not!

Can we perfectly distinguish Separable states only using LOCC?

Nonlocality without entanglement

State preparation: Easy



Randomly choose among

$$|\Psi_1\rangle_{AB} = |\psi_1\rangle_A \otimes |\phi_1\rangle_B$$
$$|\Psi_2\rangle_{AB} = |\psi_2\rangle_A \otimes |\phi_2\rangle_B$$
$$\vdots$$
$$|\Psi_k\rangle_{AB} = |\psi_k\rangle_A \otimes |\phi_k\rangle_B$$

* Ψ_j are orthogonal, but ψ_j, ϕ_j might not!

Measurement: Difficult

Perfect discrimination via $\{|\Psi_i\rangle\langle\Psi_i| = |\psi_i\rangle\langle\psi_i|_A \otimes |\phi_i\rangle\langle\phi_i|_B\}$ $\Subset$

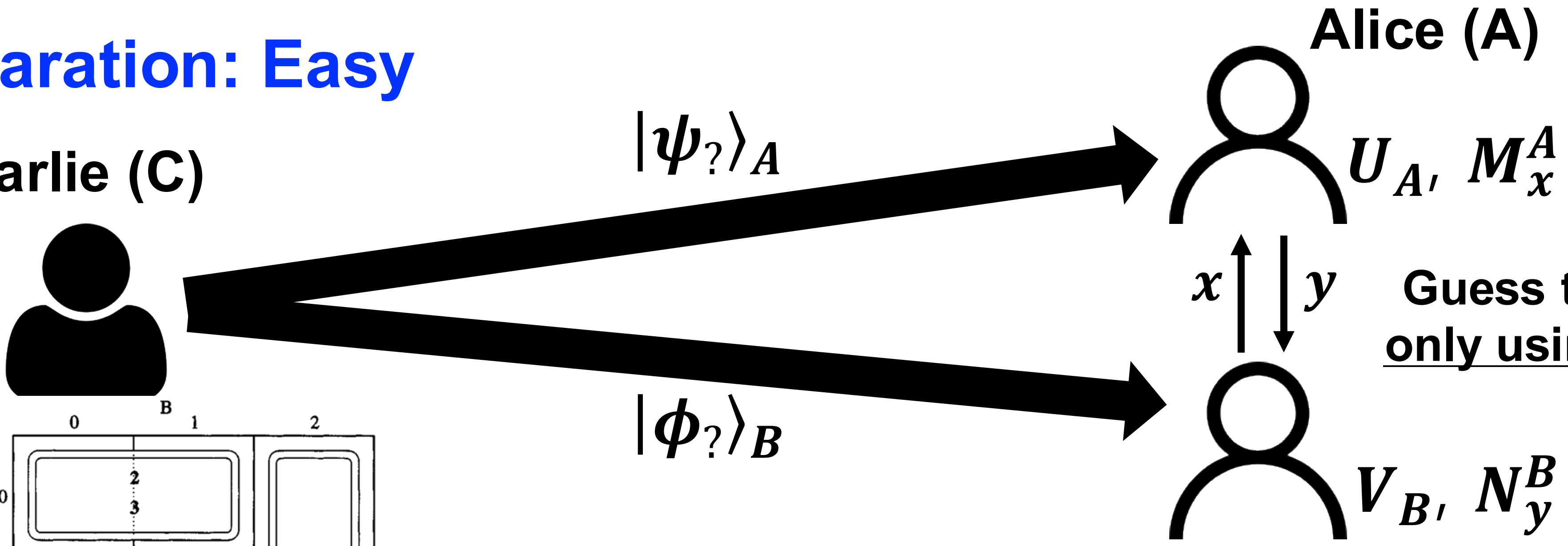
There exists a set of separable states that cannot be perfectly distinguished by LOCC! (Nonlocality without entanglement)

➲ "LOCC $\subsetneq$ SEP"

[Phys. Rev. A 59, 1070 (1999)]

Nonlocality without entanglement

State preparation: Easy



Guess the label <u>only using LOCC</u>

Measurement: Difficult

$\psi_1 = |1\rangle \otimes |1\rangle,$

$\psi_2 = |0\rangle \otimes |0+1\rangle,$

$\psi_3 = |0\rangle \otimes |0-1\rangle,$

$\psi_4 = |2\rangle \otimes |1+2\rangle,$

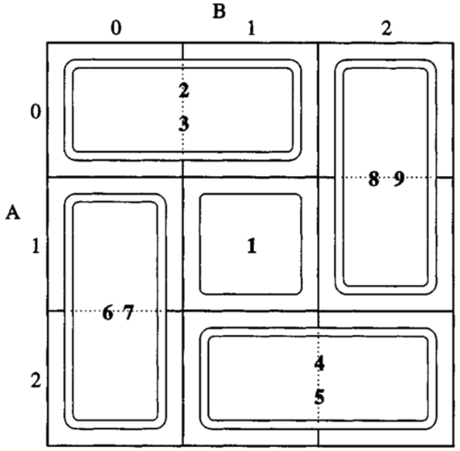
$\psi_5 = |2\rangle \otimes |1-2\rangle,$

$\psi_6 = |1+2\rangle \otimes |0\rangle,$

$\psi_7 = |1-2\rangle \otimes |0\rangle,$

$\psi_8 = |0+1\rangle \otimes |2\rangle,$

$\psi_9 = |0-1\rangle \otimes |2\rangle.$



There exists a set of separable states that <u>cannot</u> be perfectly distinguished by LOCC!
(Nonlocality without entanglement)

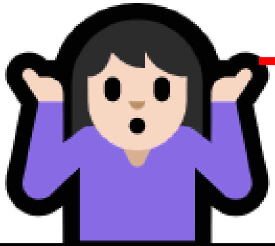
➲ "LOCC ⊊ SEP"

[Phys. Rev. A 59, 1070 (1999)]

Main question

Can we find such phenomena in other resource theories?

Yes, in quantum resource theory of magic (non-stabilizerness)



arXiv > quant-ph > arXiv:2509.25790

Quantum Physics

[Submitted on 30 Sep 2025]

~~Nonstabilizerness without Magic~~: Classically Simulatable Quantum States That Are Indistinguishable by Classically Simulatable Quantum Circuits

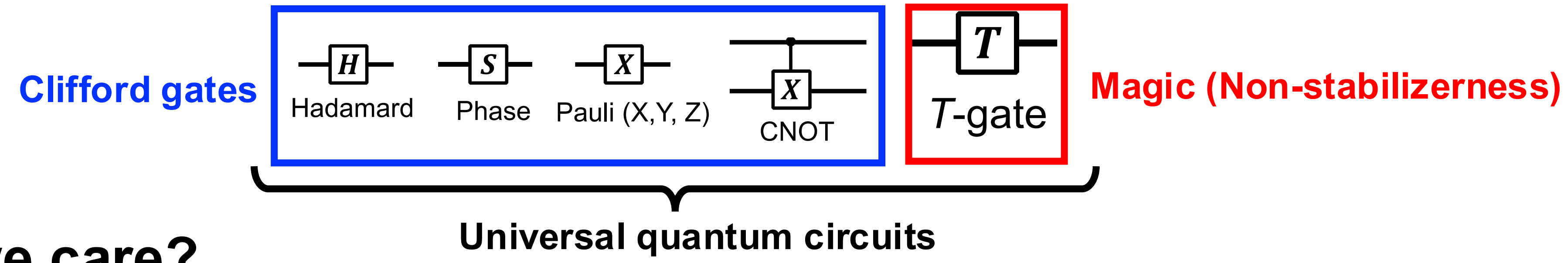
Hyukjoon Kwon

Resource theory of magic

❑ Resource theory of magic (non-stabilizerness)

★ Next talk (by Jens)

➲ **Free states**: Stabilizer states that can be generated by Clifford circuits (CNOT, S, H) on $|0\rangle^{\otimes n}$



❑ Why do we care?

➲ Classical simulability (Gottesman-Knill Theorem)

➲ Fault tolerant implementation of non-Clifford gates (Eastin-Knill theorem)

★ Rhea's talk on Tuesday

❑ Free operations

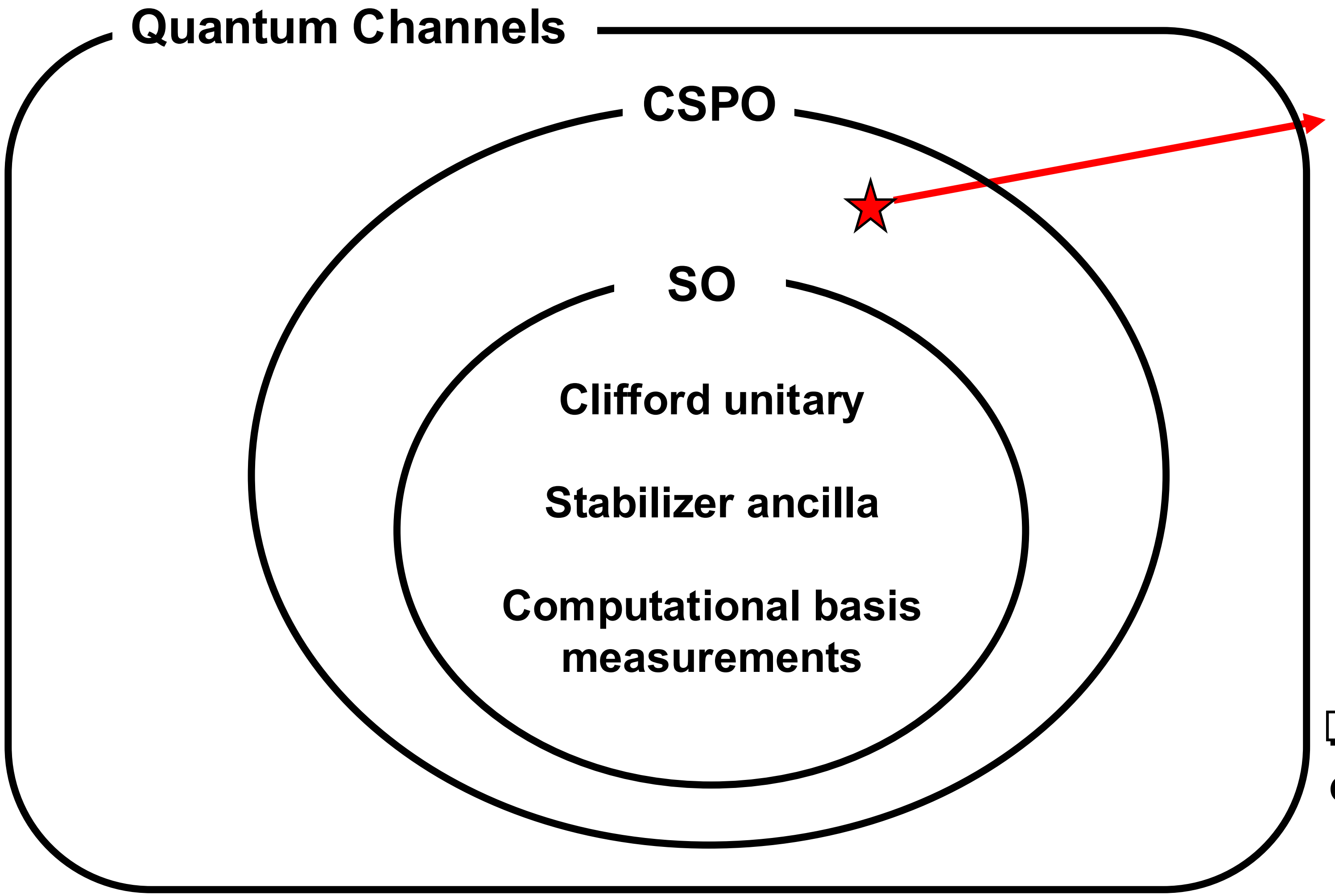
➲ **Axiomatic**: **C**ompletely **S**tabilizer **P**reserving **O**perations: $(\Phi \otimes \mathrm{id.})(\rho) \in \mathcal{F}\ \ \forall \rho \in \mathcal{F})$

➲ **Operational**: **S**tabilizer **O**perations

1) Stabilizer state preparation
2) Clifford unitary operations
3) Computational basis measurements

* Any channel that can be (adaptively) implemented by a series of these operations

Axiomatic vs. Operational free operations

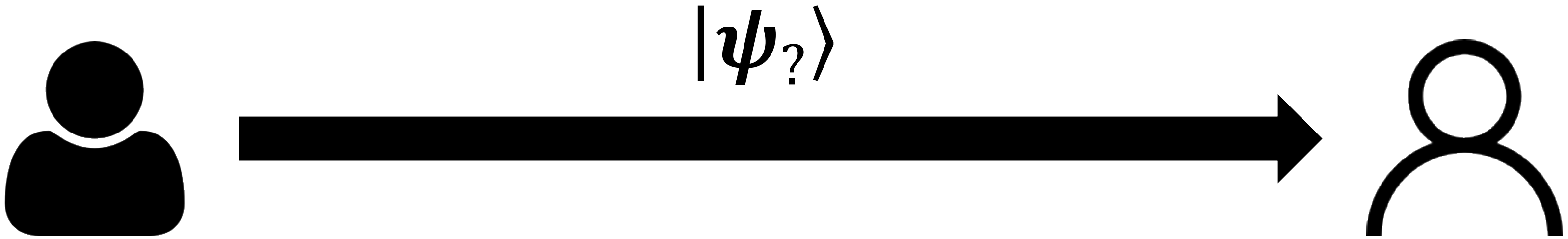


❑ **Is SO $\subsetneq$ CSPO?**
➲ **Explicit counter-example**
[Heimendahl, Heinrich, Gross (2022)]

❑ **Limitations of SO for quantum state discrimination?**
➲ [Zhu, Liu, Zhu, Wang (2024)]
* **For discriminating odd prime-dim. magic state vs. its orthogonal complement**

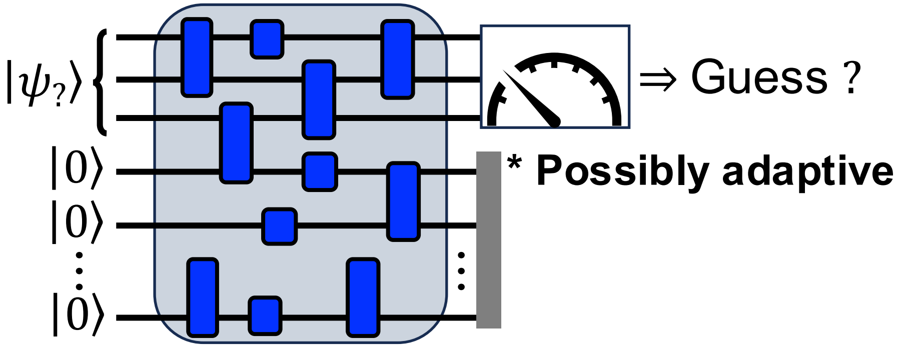
❑ **Limitation of SO for discriminating orthogonal (n-qubit) stabilizer states (This work)**

Stabilizer state discrimination via stabilizer operations

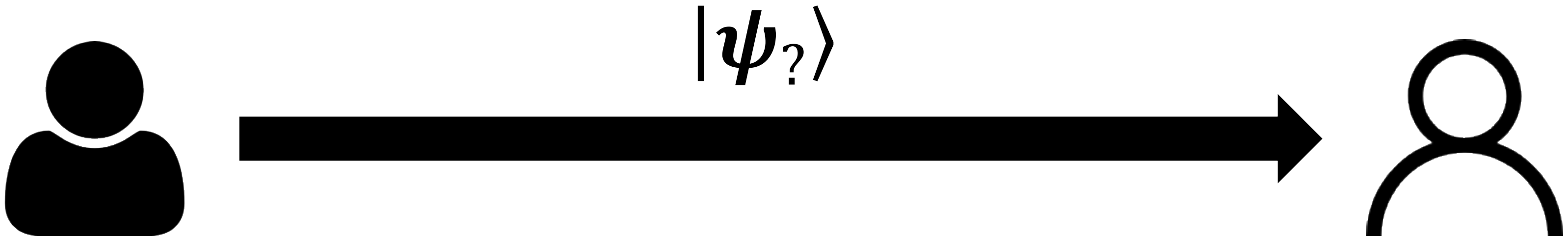


Randomly choose among stabilizer states $\{|\psi_k\rangle\}_{k=1}^{K}$

Guess the correct label using stabilizer operations



Stabilizer state discrimination via stabilizer operations



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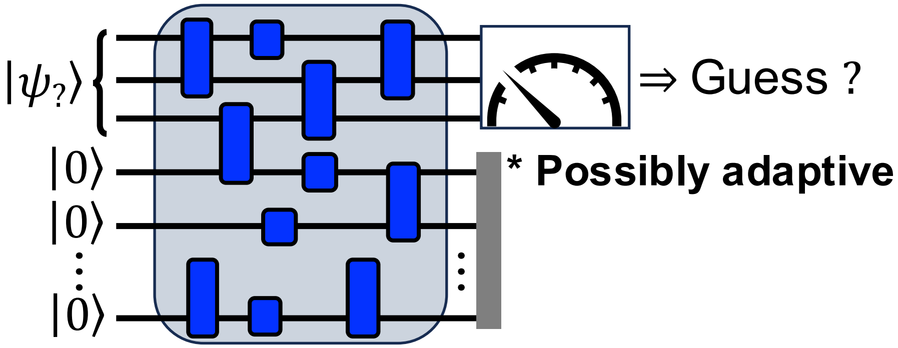
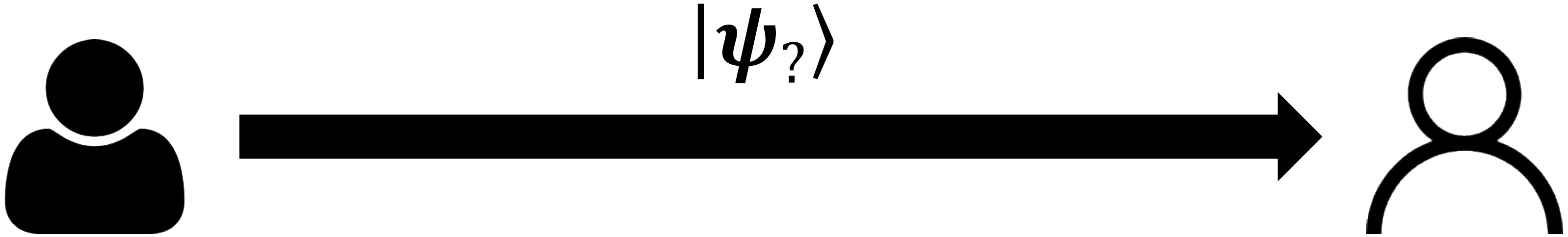


Can we perfectly distinguish stabilizer states only using stabilizer operations?

[Main result]

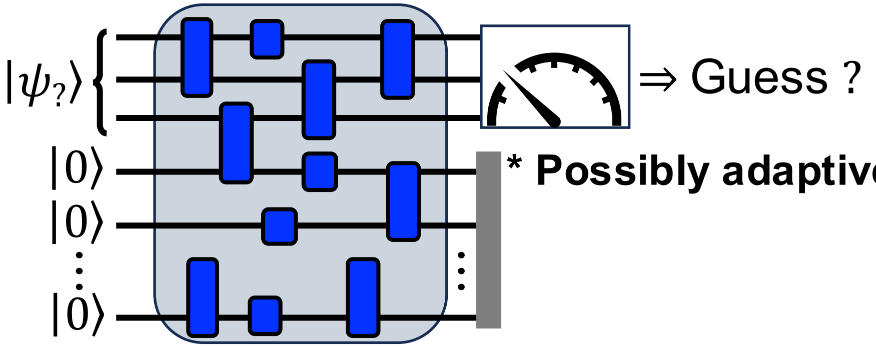
There exists a set of mutually orthogonal stabilizer states than cannot be perfectly discriminated by stabilizer operations

Stabilizer state discrimination via stabilizer operations



Randomly choose among stabilizer states $\{|\psi_k\rangle\}_{k=1}^{K}$

Guess the correct label using stabilizer operations



$$|\psi_1\rangle = |?\rangle|1\rangle|0\rangle$$
$$|\psi_2\rangle = |0\rangle|+\rangle|1\rangle$$
$$|\psi_3\rangle = |1\rangle|0\rangle|+\rangle$$
$$|\psi_4\rangle = |?\rangle|1\rangle|0\rangle$$
$$|\psi_5\rangle = |0\rangle|-\rangle|1\rangle$$
$$|\psi_6\rangle = |1\rangle|0\rangle|-\rangle$$

➲ **Pure state example with minimum # of qubits**

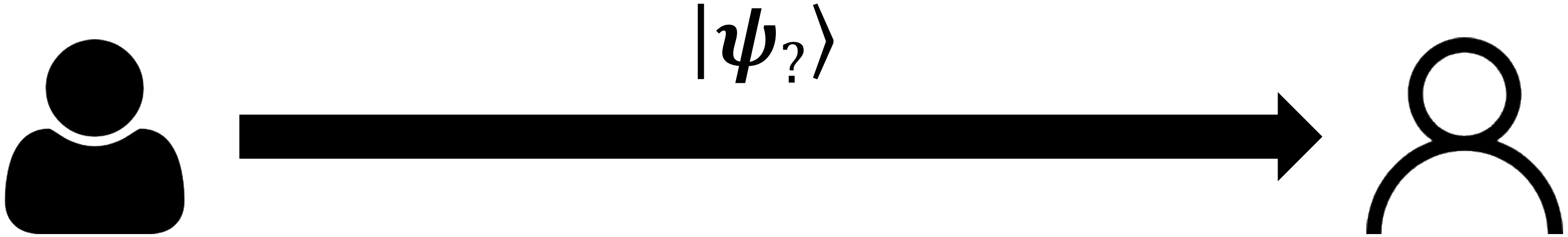
(c.f.) For mixed states, two-qubit (still orthogonal) stabilizer states are enough

$$\rho_0 = \frac{1}{2}(|0\rangle\langle 0| \otimes |+\rangle\langle +| + |+\rangle\langle +| \otimes |0\rangle\langle 0|)$$
$$\rho_1 = \frac{1}{2}(|1\rangle\langle 1| \otimes |1\rangle\langle 1| + |-\rangle\langle -| \otimes |-\rangle\langle -|)$$

➲ **Proof idea: 1) Any z-measurements followed by Clifford gates can be expressed as $\Pi_{\pm}^{P} = \frac{\mathbb{I} \pm P}{2}$ with some Pauli operator P**

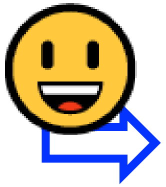
2) For any $\Pi_{\pm}^{P}$, there exists a pair of $\{|\psi_i\rangle, |\psi_j\rangle\}$ with non-vanishing overlap

Success probability bound for stabilizer operations



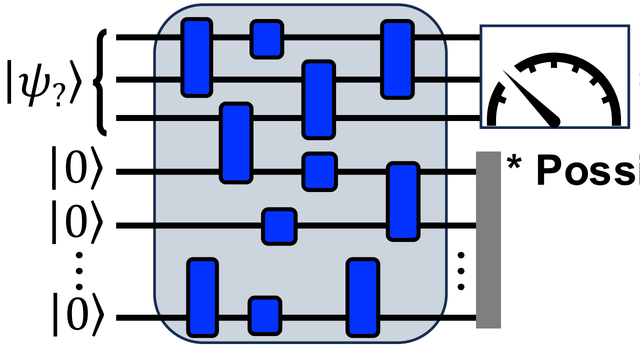
Randomly choose among stabilizer states $\{|\psi_k\rangle\}_{k=1}^{K}$

$$|\psi_1\rangle = |+\rangle|1\rangle|0\rangle$$
$$|\psi_2\rangle = |0\rangle|+\rangle|1\rangle$$
$$|\psi_3\rangle = |1\rangle|0\rangle|+\rangle$$
$$|\psi_4\rangle = |-\rangle|1\rangle|0\rangle$$
$$|\psi_5\rangle = |0\rangle|-\rangle|1\rangle$$
$$|\psi_6\rangle = |1\rangle|0\rangle|-\rangle$$



$$p_{SO}^{succ} = \frac{5}{6}$$

Guess the correct label using stabilizer operations



*** Possibly adaptive**

❑ **What is the best strategy with SO?**

(Good News) SO has a less complicated structure than LOCC

$$\Phi_{SO}(\rho) = \mathrm{Tr}_{anc}\left[\sum_i U_i (P_i \rho P_i \otimes |0\rangle_{anc}\langle 0|) U_i^\dagger\right]$$ [Heimendahl, Heinrich, Gross (2022)]

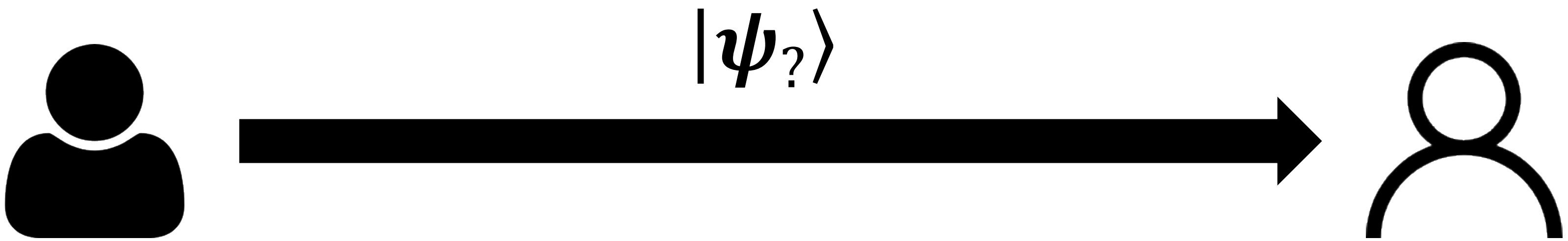
where P_i: mutually orthogonal stabilizer code projectors & U_i: Clifford unitaries

* We only need n rounds of binary measurements for n-qubit SO

(Bad News) Need to search double exponential ($\sim 4^{2^{n+1}-n-2}$) possibilities

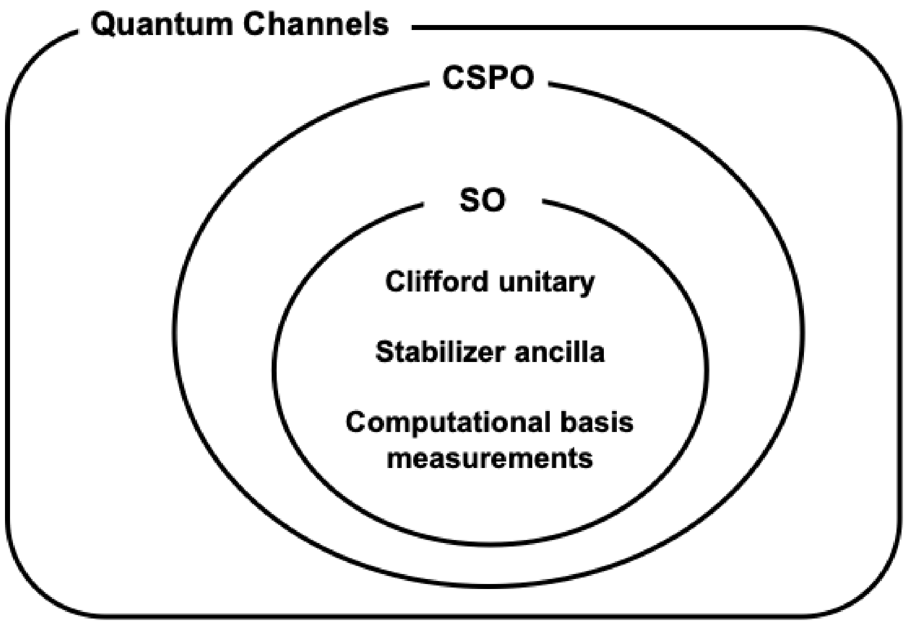
* $n = 3$ is the maximum # of qubits that we can run the algorithm

Discrimination beyond stabilizer operations



Randomly choose among stabilizer states $\{|\psi_k\rangle\}_{k=1}^K$

Guess the correct label using stabilizer operations

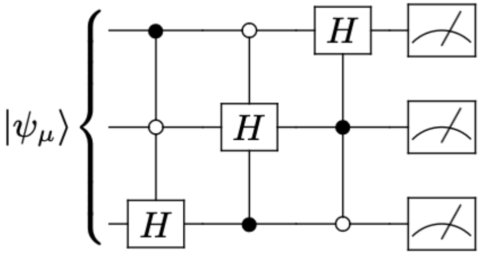


❑ Perfect discrimination is possible via projection $\Pi_{\psi_k} = |\psi_k\rangle\langle\psi_k|$

➲ $\{E_j\} = \{\Pi_{\psi_k}\}_{k=1}^K \cup \left(\mathbb{I} - \sum_{k=1}^K \Pi_{\psi_k}\right) \in \mathrm{CSPO}$

$\therefore \mathrm{SO} \subsetneq \mathrm{CSPO}$

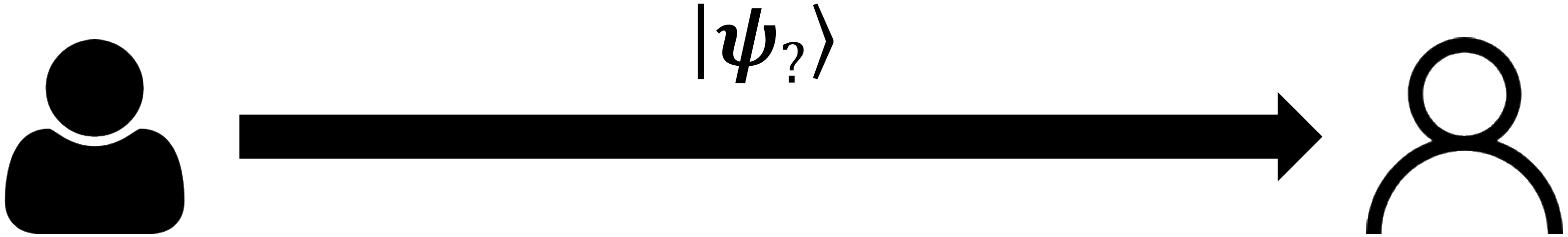
❑ Non-Clifford circuits for **perfect discrimination**



Input	Output
$\|+\rangle\|1\rangle\|0\rangle$	$\|0\rangle\|1\rangle\|0\rangle$
$\|0\rangle\|+\rangle\|1\rangle$	$\|0\rangle\|0\rangle\|1\rangle$
$\|1\rangle\|0\rangle\|+\rangle$	$\|1\rangle\|0\rangle\|0\rangle$
$\|-\rangle\|1\rangle\|0\rangle$	$\|1\rangle\|1\rangle\|0\rangle$
$\|0\rangle\|-\rangle\|1\rangle$	$\|0\rangle\|1\rangle\|1\rangle$
$\|1\rangle\|0\rangle\|-\rangle$	$\|1\rangle\|0\rangle\|1\rangle$

(c.f.) Single Toffoli gate + Clifford circuits is enough when using adaptive strategies

Discrimination beyond stabilizer operations



Randomly choose among stabilizer states $\{|\psi_k\rangle\}_{k=1}^{K}$

Guess the correct label using stabilizer operations

❑ What is the magic cost for perfect discrimination?

➲ How many T-states should be appended to reach $p_{\mathrm{SO}}^{succ}\left(\{|\psi_k\rangle \otimes |T\rangle^{\otimes t}\}_{k=1}^{K}\right) = 1$

➲ However, fully searching $n+t$ qubit SO is still challenging

➲ Alternatively, one can take entropic analysis for first few steps

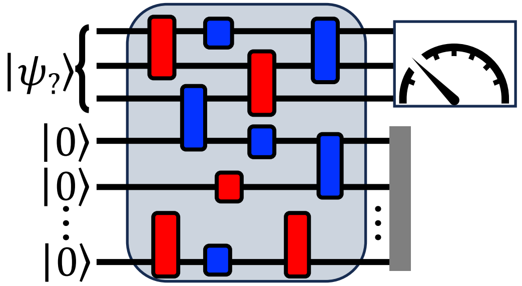
Target label (key) → K

$$I(K\!:\!AB) \le I(K\!:\!A) + \sum_{a\in A} p(a)\,\chi(p(k|a), \psi_k^a)$$

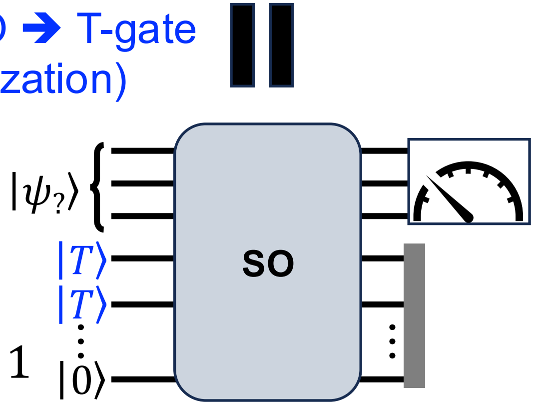
AB: All possible later outcomes; A: First measurement outcomes; $\chi(p(k|a), \psi_k^a)$: Holevo information of the remaining states

✓ For the case with SO, we have $I(K\!:\!AB) \le \log_2 6 - \frac{1}{3} \Rightarrow H(K|AB) \ge \frac{1}{3} > 0$

✓ Even with a single T-state, $H(K|AB) > 0$ ➔ 1 T-gate/state is not sufficient for $p^{succ} = 1$



T-state + SO ➔ T-gate
(Gadgetization)



Implications & applications

No Cloning theorem via restricted operations

❑ **No-Cloning theorem:** $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \rightarrow |\psi\rangle \otimes |\psi\rangle$ **is not possible for unknown** $\alpha, \beta \in \mathbb{C}$

"Randomly choose"
(State preparation)

"Guess the correct label"
(via Measurement)

$|\psi_1\rangle = |+\rangle|1\rangle|0\rangle$

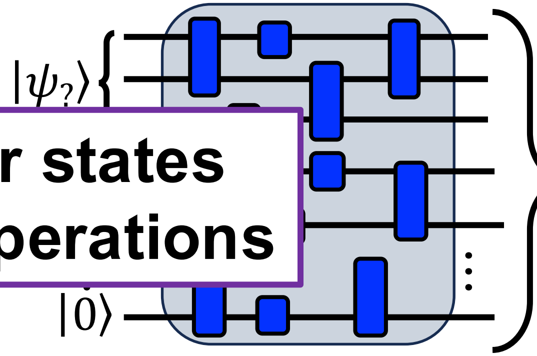
$|\psi_2\rangle = |0\rangle|$

$|\psi_3\rangle = |1\rangle|$

$|\psi_4\rangle = |-\rangle|1\rangle|0\rangle$

$|\psi_5\rangle = |0\rangle|-\rangle|1\rangle$

$|\psi_6\rangle = |1\rangle|0\rangle|-\rangle$



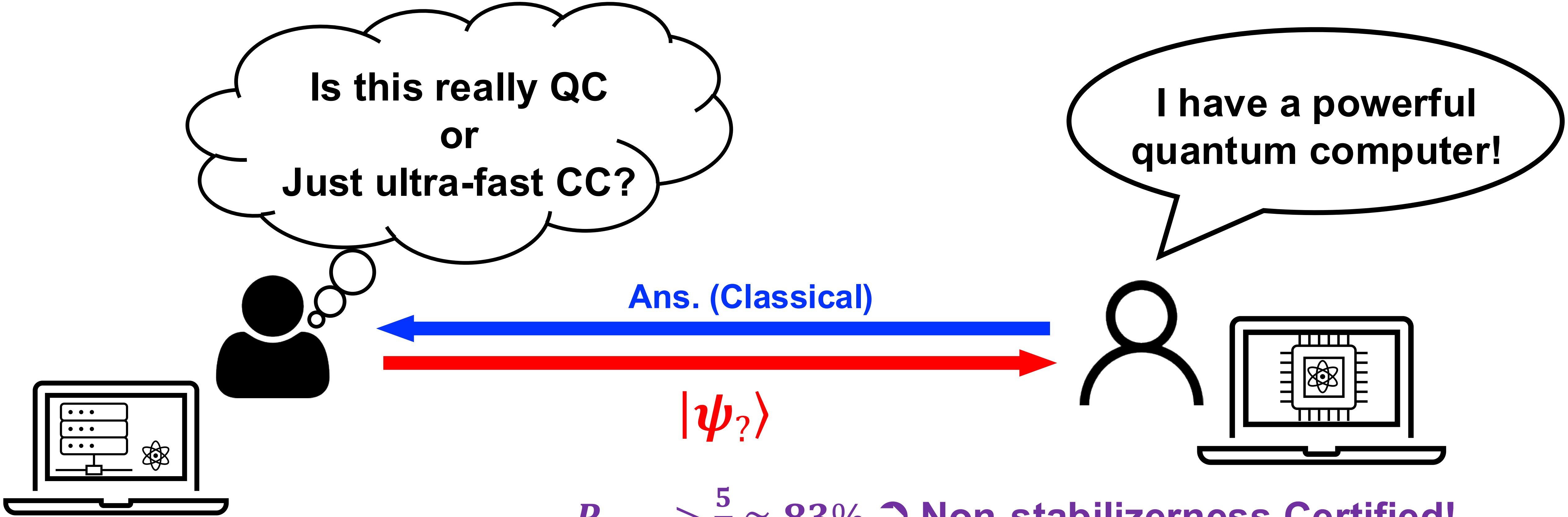
Stabilizer Operations

$\Rightarrow p_{SO}^{succ} = 1$

Cloning of orthogonal stabilizer states is still impossible by stabilizer operations

Repeat this many time ➲ **One can break the bound of the success probability $\leq 5/6$**

Certifying quantum computers



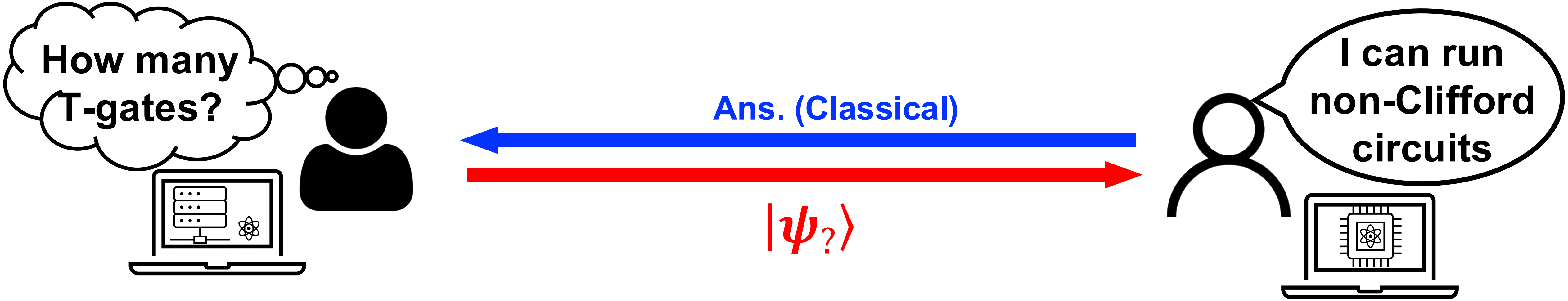
Only low-cost & product stabilizer states are used!

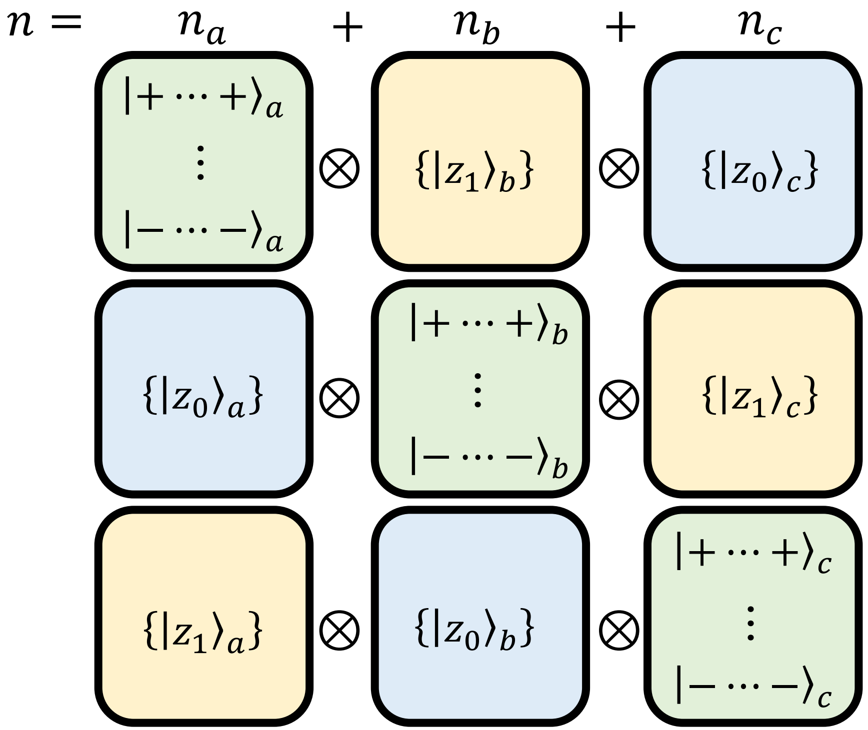
$P_{\text{succ}} > \frac{5}{6} \approx 83\%$ ➲ **Non-stabilizerness Certified!**

(e.g.) N = 1000 trial with 900 success (≈7% higher)
➲ Nonstabilizerness is certified with ≈5.6σ

➲ **A small number of checks is enough!**
➲ **Unconditional (no computational complexity assumptions)!**

n-qubit generalization





❑ We take a highly non-linear Boolean function $f_{a,b,c}$ and define

$$\left\{\left|z^{a,b,c}_{s=0,1}\right\rangle_{a,b,c}\right\} \text{ s.t. } f_{a,b,c}\left(z^{a,b,c}_{s}\right) = s \in \{0,1\}$$

➲ (Conjecture 1) $p^{succ}_{\mathrm{SO}} = \frac{2+2^{-n_{\min}}}{3}$ ➔ $p^{succ}_{\mathrm{SO}} \xrightarrow{n\to\infty} \frac{2}{3}$

(∵ Exponentially many states to be discriminated in each sector)

➲ (Conjecture 2) Perfect discrimination requires

$$U_f(|z\rangle|y\rangle) = |z\rangle|y \oplus f(z)\rangle.$$

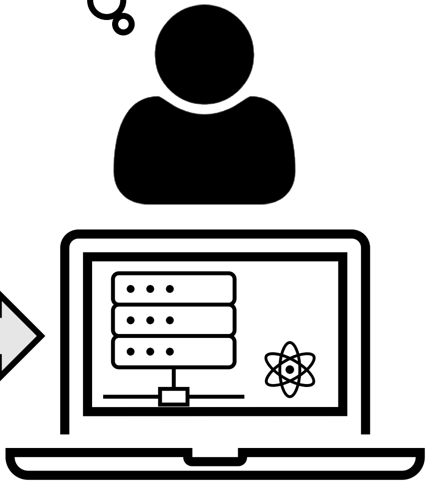
If this is true, **"(non-adaptive) T-gate cost = $\Theta(n)$"**

[Gosset, Kothari, Zhang, arXiv:2510.07223]

Data hiding in stabilizer states

I'd like to hide data

"But I can only prepare stabilizer states"



1010
1010

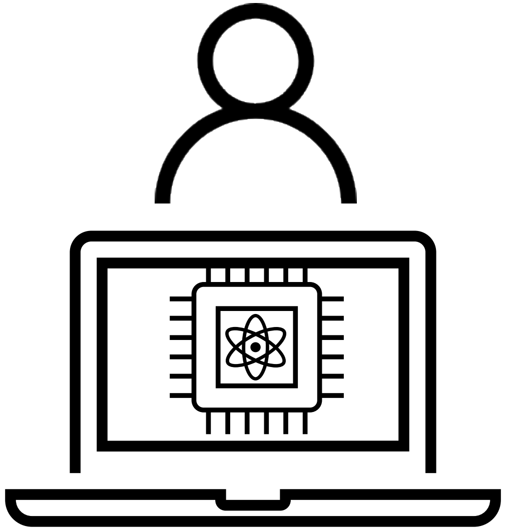
$|\psi_1\rangle = |+\rangle|1\rangle|0\rangle$
$|\psi_2\rangle = |0\rangle|+\rangle|1\rangle$
$|\psi_3\rangle = |1\rangle|0\rangle|+\rangle$
} "0"

$|\psi_4\rangle = |-\rangle|1\rangle|0\rangle$
$|\psi_5\rangle = |0\rangle|-\rangle|1\rangle$
$|\psi_6\rangle = |1\rangle|0\rangle|-\rangle$
} "1"

$|\psi_1\rangle|\psi_2\rangle|\psi_3\rangle|\psi_6\rangle \cdots$

"0" "0" "0" "1"

❑ Difficult to decode
➲ p_{SO}^{succ} is still $\frac{5}{6}$
(data hiding ratio = $\frac{2}{3}$)



1010
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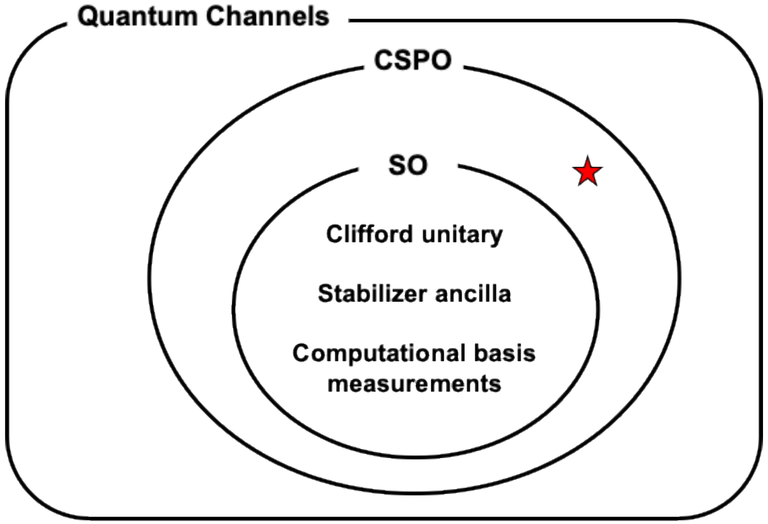
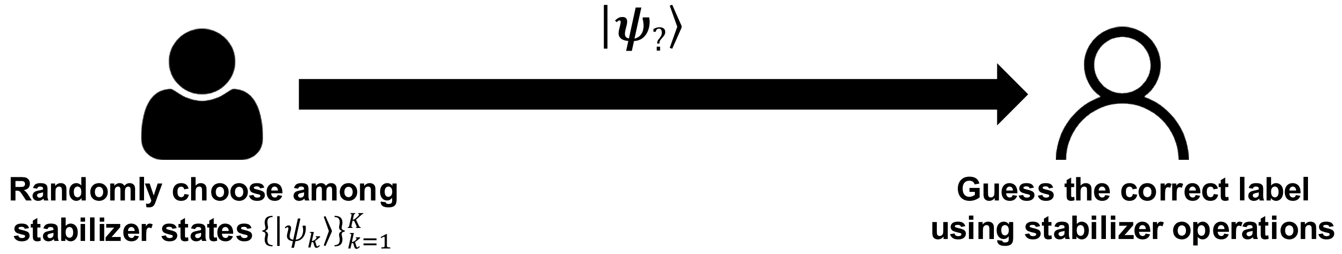
❑ Much easier to prepare and store (error correction)

❑ Decoding this information is possible only with
(fault-tolerant & many) non-stabilizer quantum circuits
(which might take very long time to build)

Remarks & Open problems





[Main result]

There exists a set of mutually orthogonal stabilizer states than cannot be perfectly discriminated by stabilizer operations

For more information: arXiv:2509.25790

❑ Open problems

- ➲ Can we calculate/bound success probability and magic cost more efficiently?
- ➲ d-dimensional generalization? (Probably, yes)
- ➲ Relationship with entanglement ($p_{\mathrm{LOCC}}^{succ} < 1 \Leftrightarrow p_{\mathrm{SO}}^{succ} < 1$ for separable stabilizer states?)
  - ✓ 2 qubit mixed & 3 qubit pure state examples also feature “nonlocality without entanglement.”
- ➲ Can we construct bound magic states with this property?
- ➲ Any relation to (provable) quantum computational advantage?

❑ More general questions regarding quantum resource theory

- ➲ Under which general conditions, free operations cannot discriminate free states?
- ➲ Are there some other tasks that only involve free states, but not possible by only free operations?

- James Moran, Spiros Kechrimparis, Hyukjoon Kwon
  Near-optimal coherent state discrimination via continuously labelled non-Gaussian measurements

- Adam G Hawkins, Hannah McAleese and Mauro Paternostro
  Necessary and sufficient resources for quantum telecloning

- Minjeong Song, Hyukjoon Kwon, Valerio Scarani
  Exact and approximate conditions of tabletop reversibility: when is Petz recovery cost-free?

Thank you

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