Dynamical Landauer Principle

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Leverhulme Trust

Dynamical Landauer Principle

Chung-Yun Hsieh, *Phys. Rev. Lett.* **134**, 050404 (2025)

Chung-Yun Hsieh, *Phys. Rev. A* **111**, 022207 (2025)

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What is the Landauer Principle

?

What is the Landauer Principle

?

What can we do with information

V

V

V

Gas

V

V

Isothermal Expansion

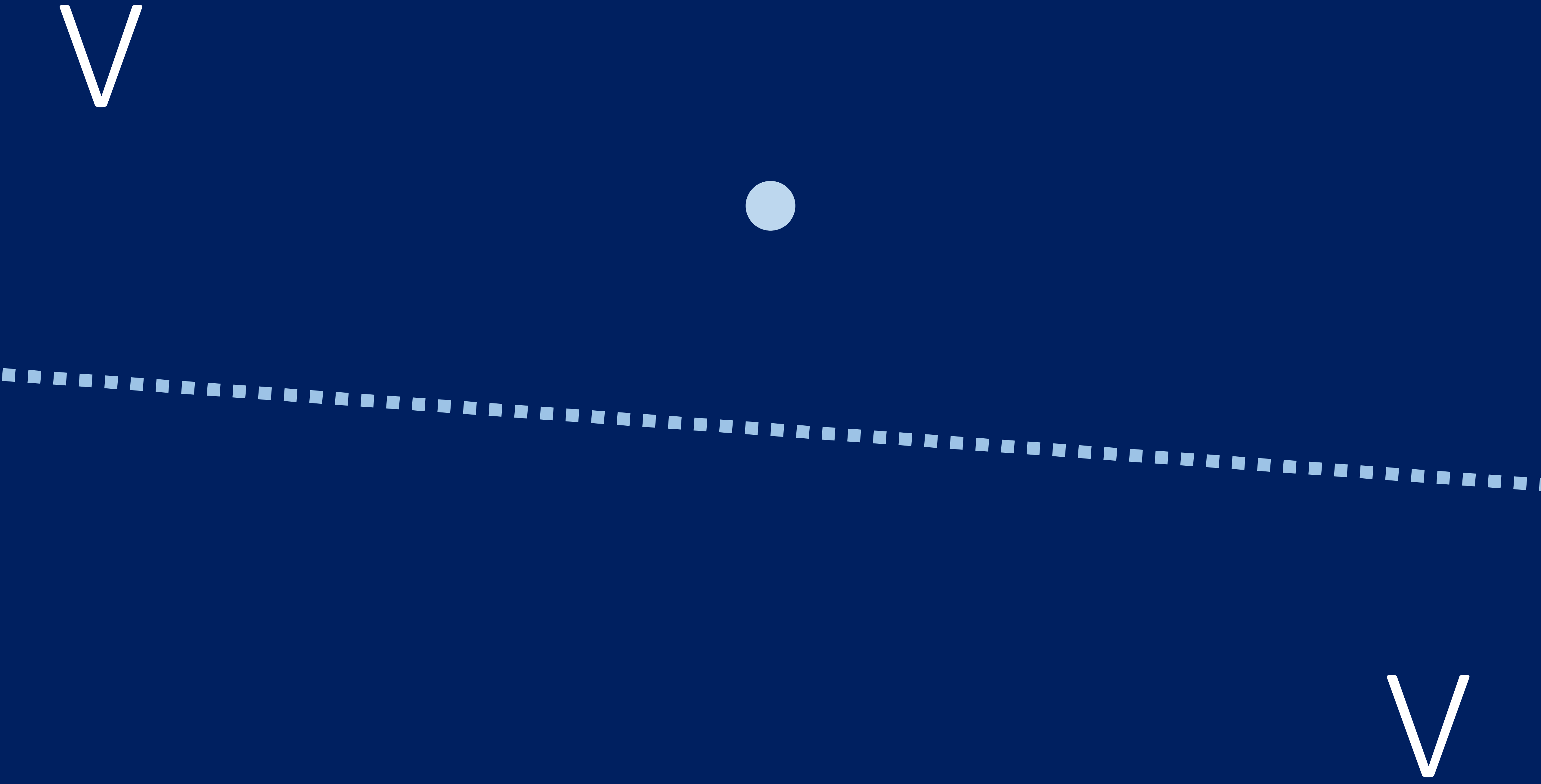
V

$$k_B T \ln 2$$

$k_B T \ln 2$

Szilard Engine

Gas


V
V

V

Isothermal Compression

V

$$-k_B T \ln 2$$

$$-k_B T \ln 2$$

Landauer's Principle

Preparing n classical bits comes with costing at least $n \times k_B T \ln 2$ energy

Landauer's Principle

R. Landauer, *IBM J. Res. Dev.* **5**, 183 (1961)

Transmitting n classical bits comes with

Transmitting n classical bits comes with

what thermodynamic criteria

Sending classical bits quantum-mechanically

A

B

A

B


A
M
B
M

A

B

A

B

Q system

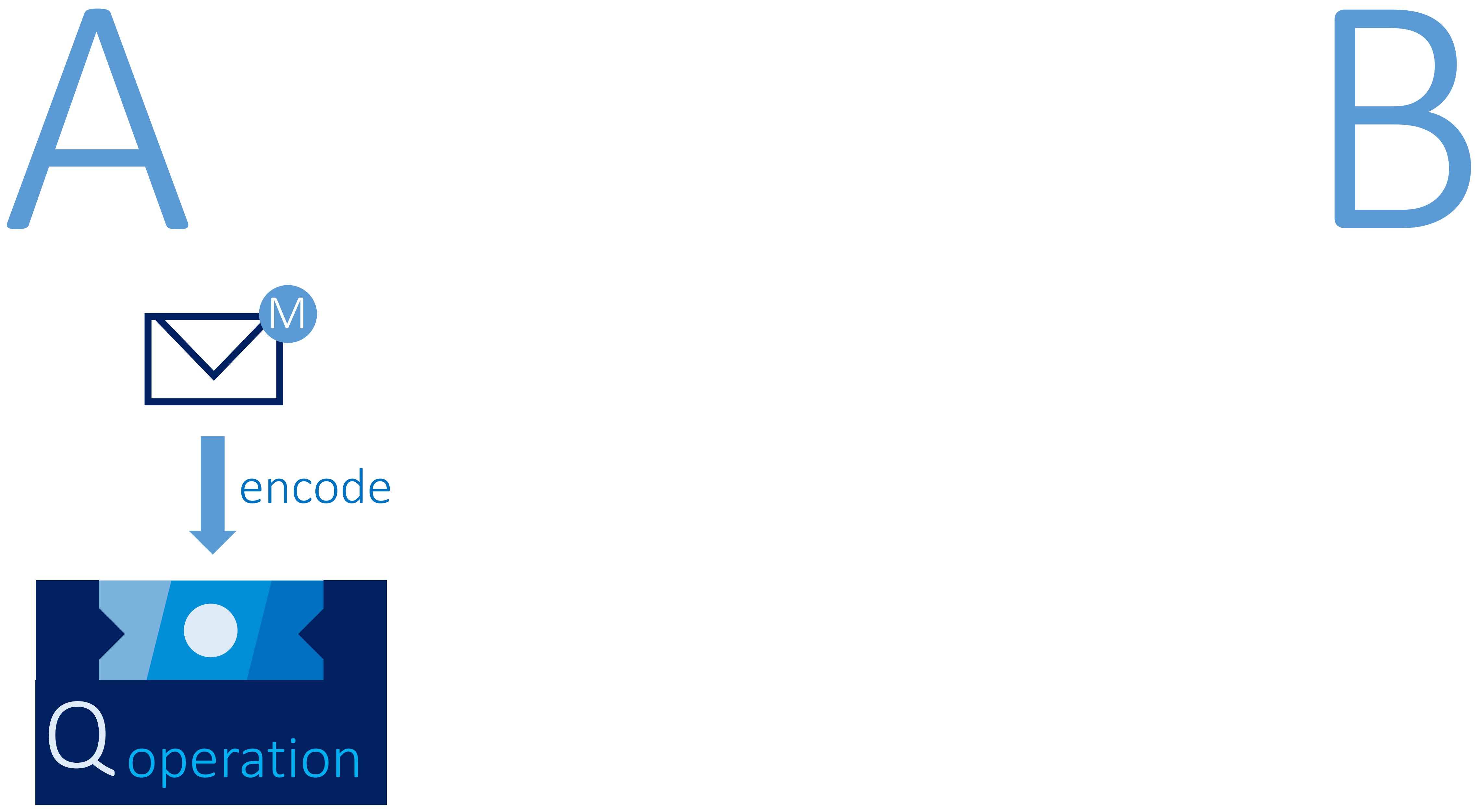

A
B
M
Q operation

A

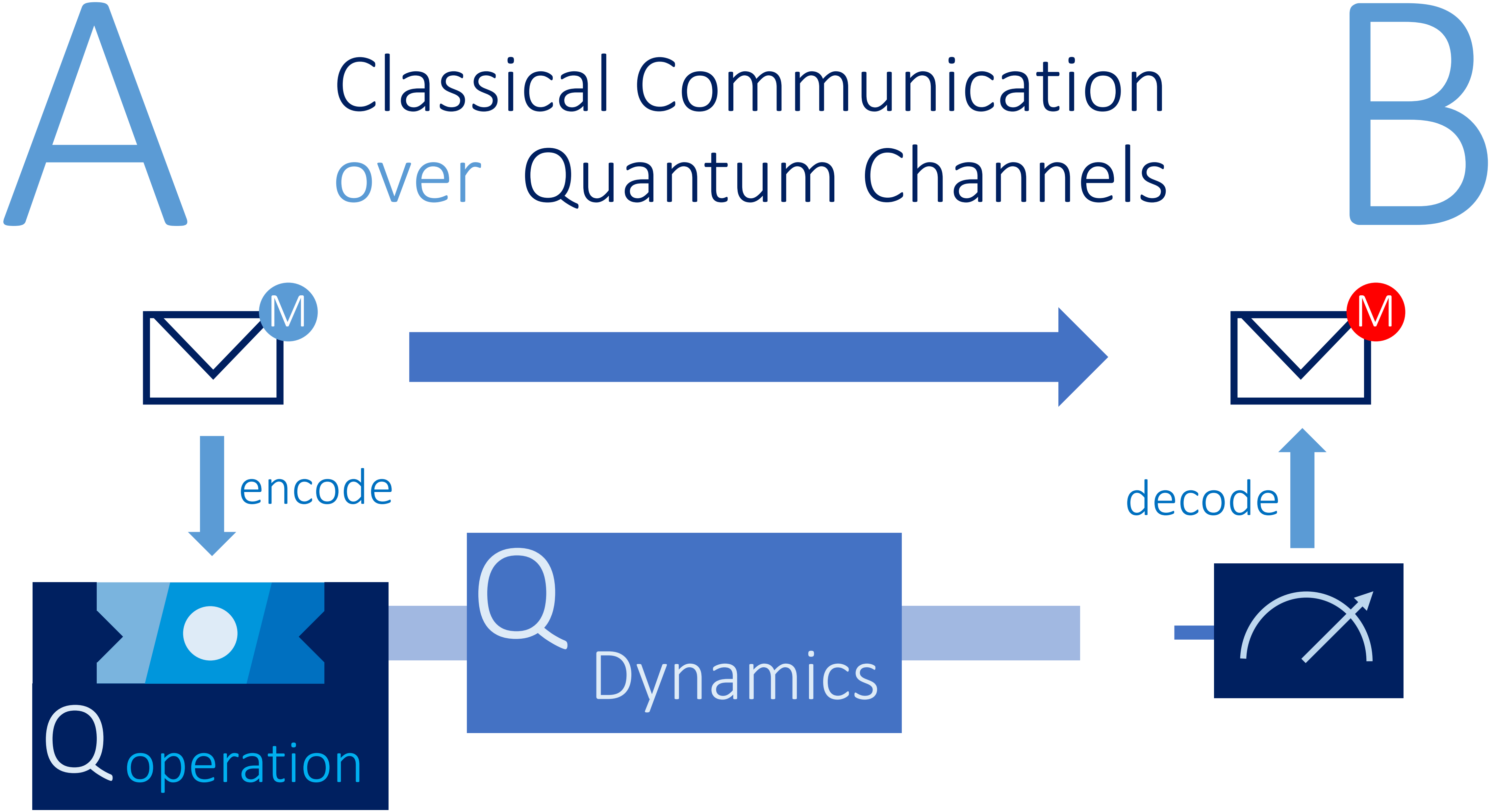
B

M

Q operation

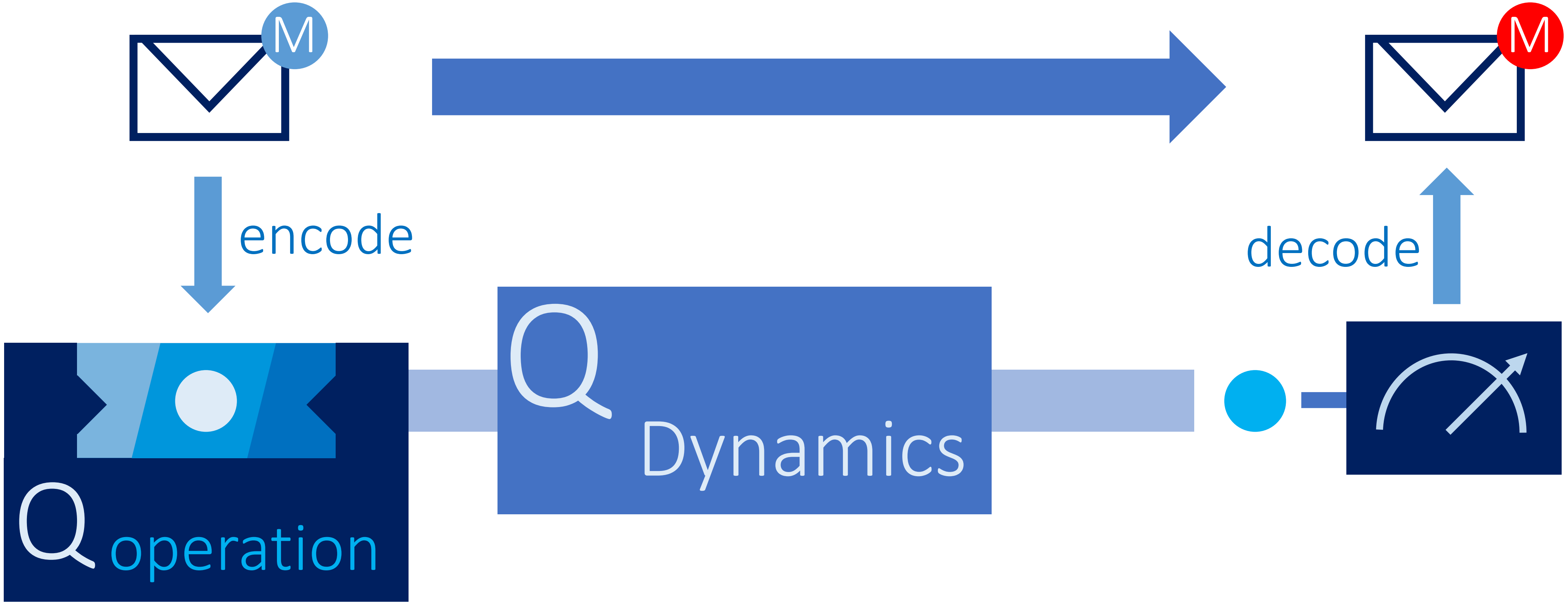

A
B
M
encode
Q operation

Classical Communication over Quantum Channels



Classical Capacity

C = highest amount of transmitted bits

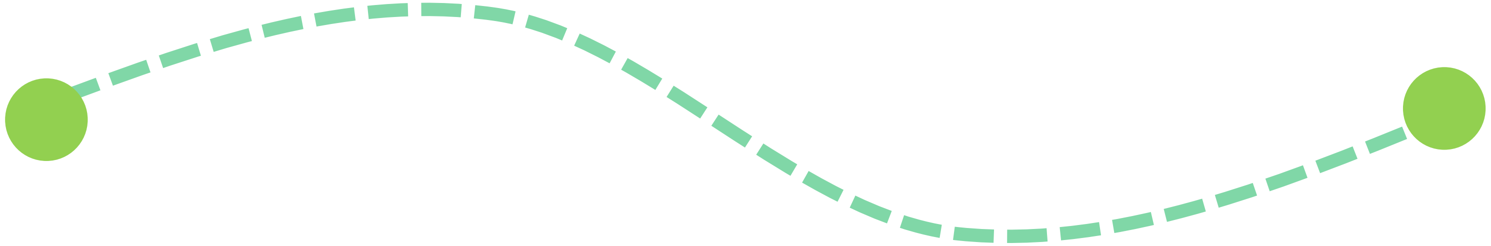


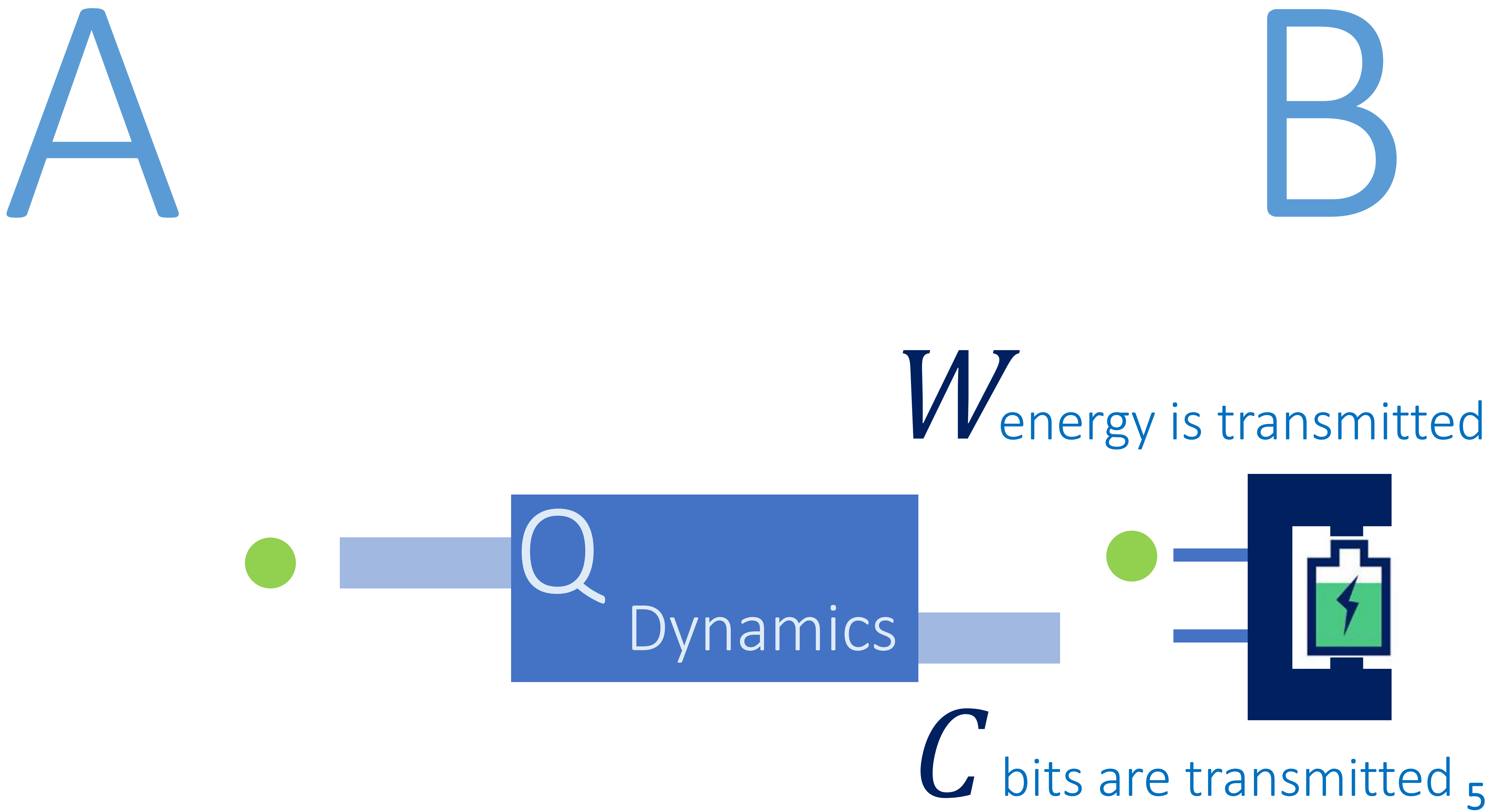
Energy transmitted by quantum dynamics

A

B

$$\Phi_{AB} = \sum_{m=0}^{M-1} \frac{1}{M} |mm\rangle\langle mm|_{AB}$$




A
B
W energy is transmitted
Q
Dynamics
C bits are transmitted

$$(k_B T \ln 2)\, C = W$$

Transmitting n classical bits comes with transmitting at least $n \times k_B T \ln 2$ energy

W energy is transmitted

C bits are transmitted

Transmitting n classical bits comes with what thermodynamic criteria

Transmitting n classical bits comes with transmitting at least $n \times k_B T \ln 2$ energy

$$(k_B T \ln 2)\, C = W$$

Recall that...

Preparing n classical bits comes with costing at least $n \times k_B T \ln 2$ energy

Landauer's Principle

R. Landauer, *IBM J. Res. Dev.* **5**, 183 (1961)

Dynamical Landauer Principle

Transmitting n classical bits comes with transmitting at least $n \times k_B T \ln 2$ energy

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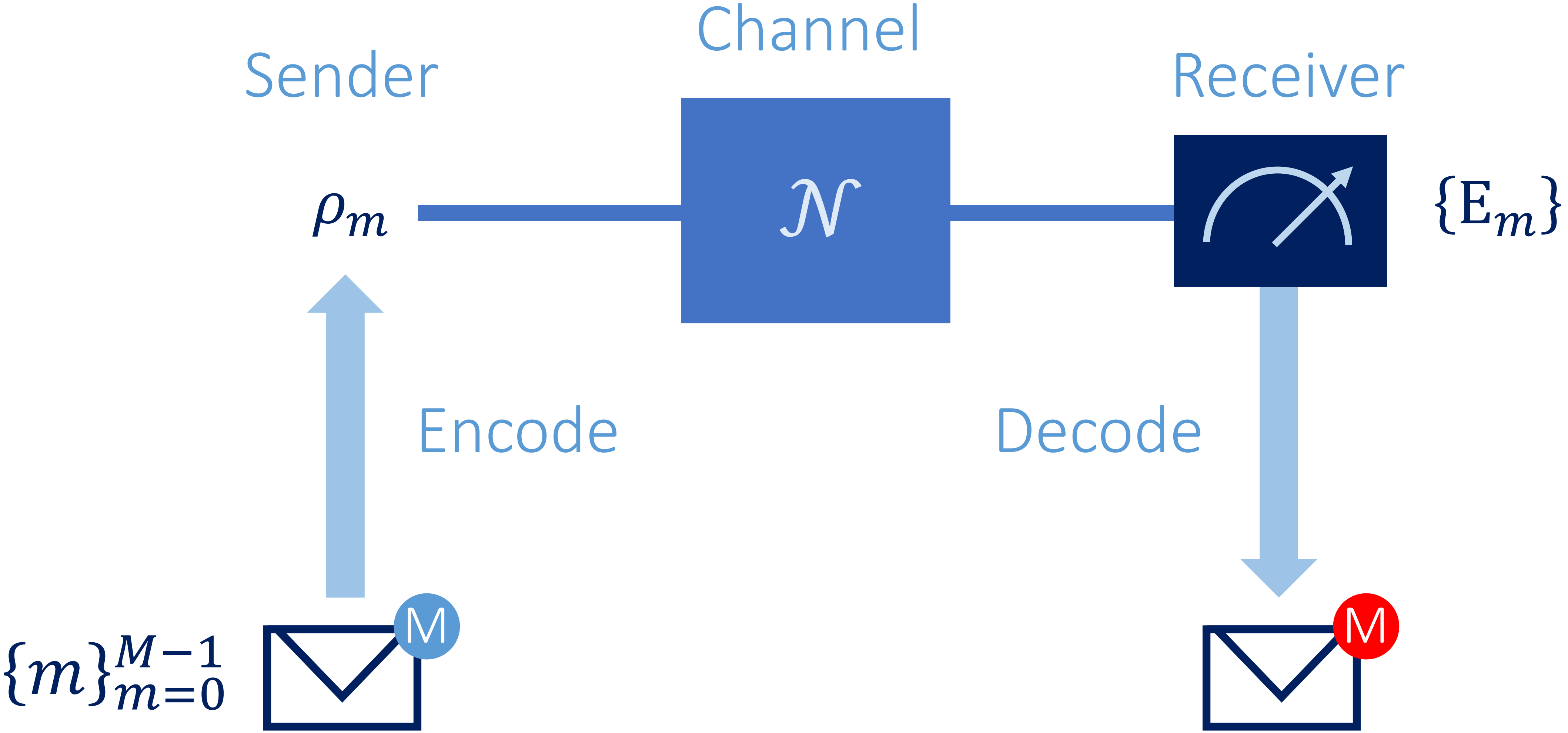
Appendix
Classical Communication via Quantum Channels

Technical Details



Technical Details

Classical capacity of $\mathcal{N}$ reads $C(\mathcal{N}) \coloneqq \lim_{\epsilon\to 0} \lim_{k\to\infty} \frac{C_{(1)}^{\epsilon}(\mathcal{N}^{\otimes k})}{k}$

One-shot classical capacity of $\mathcal{N}$ with error ϵ is defined by...

$$C_{(1)}^{\epsilon}(\mathcal{N}) \coloneqq$$

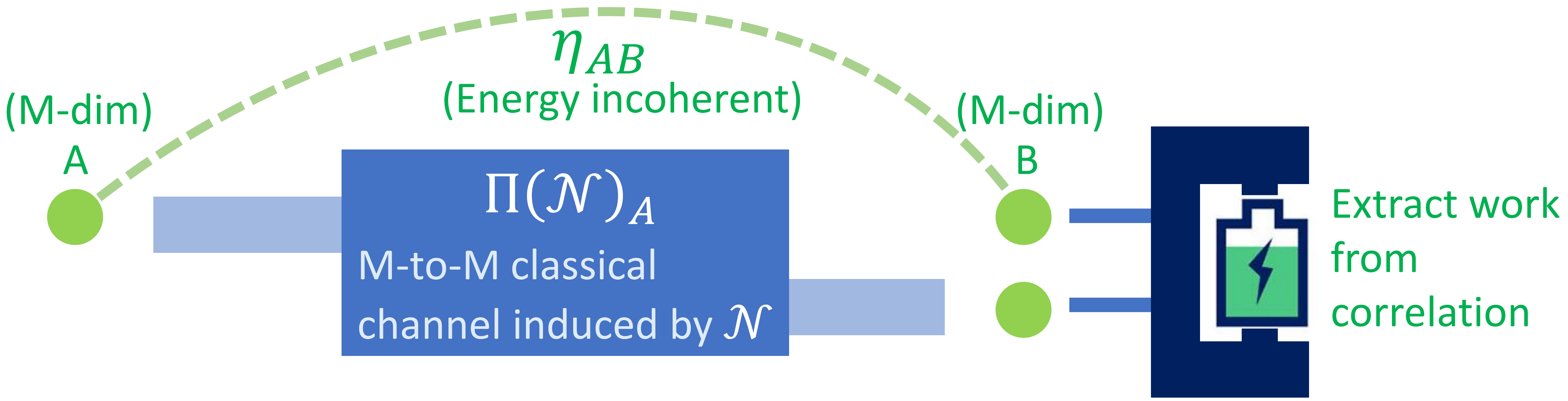
$$\max \log_2 \left\{ M \;\middle|\; \exists \{\rho_m\}, \{E_m\} \longrightarrow \frac{1}{M} \sum_{\mathrm{m=0}}^{\mathrm{M-1}} \mathrm{tr}[E_m \mathcal{N}(\rho_m)] \geq 1 - \epsilon \right\}$$

L. Wang and R. Renner, *Phys. Rev. Lett.* **108**, 200501 (2012)

Appendix
Energy Transmission via Quantum Channels

Technical Details

Asymptotic form is given by $W(\mathcal{N}) := \lim_{\epsilon \to 0} \lim_{k \to \infty} \frac{W^{\epsilon}_{(1)}(\mathcal{N}^{\otimes k})}{k}$



One-shot genuinely transmitted energy of $\mathcal{N}$ with error ϵ is defined by…

$$W^{\epsilon}_{(1)}(\mathcal{N}) := \max\{W^{\epsilon}_{\mathrm{corr},(1)}[\Pi(\mathcal{N})_A \otimes I_B](\eta_{AB}) \mid M, \eta_{AB}, \Pi(\mathcal{N})_A\}$$

J. Åberg, *Nat. Commun.* **4**, 1925 (2013)

Appendix
Formal Statement of Main Result

Main Theorem

Technical Details

Consider a background temperature $0 < T < \infty$ and a channel $\mathcal{N}$.

For error parameters $0 < \delta \leq \omega < \epsilon \leq 1 - 1/\sqrt{2}$, we have that

$$W_{(1)}^{\omega}(\mathcal{N}) - k_B T \ln \frac{4\epsilon}{(\epsilon - \omega)^2 (1 - \omega)} \leq (k_B T \ln 2) C_{(1)}^{\epsilon}(\mathcal{N}) \leq W_{(1)}^{\epsilon + \delta}(\mathcal{N})$$

In the iid limit, we have

$$W(\mathcal{N}) = (k_B T \ln 2) C(\mathcal{N})$$