MAGIC, MUGGLES, MEASUREMENT

A MEDITATION ON QUANTUM RESOURCES IN THREE READINGS

JENS EISERT, FU BERLIN

TOKYO, MARCH 2026

- What makes a quantum computer **powerful?**


• Look at three questions on quantum resources
• Is there more?
• What is the role of the observer?
• Where is the fine line?
QUANTUM RESOURCES
CLASSICAL

MEASUREMENT AS A RESOURCE

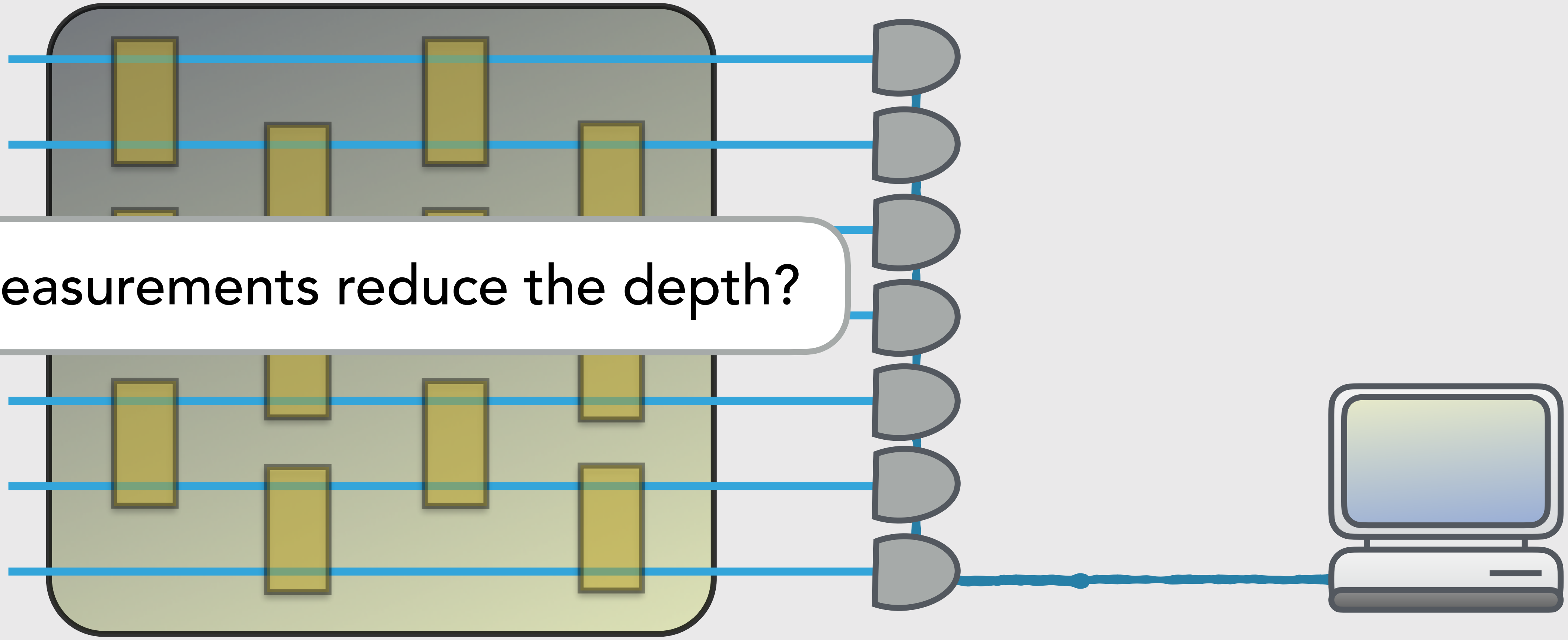
Cao, Eisert, Phys Rev Lett 136, 080601 (2026)
Liu, Ye, Cai, Eisert, arXiv:2602.22126 (2026)
Cao, Tang, Eisert, in preparation (2026)

MID-CIRCUIT MEASUREMENTS AS A RESOURCE

- How powerful are **mid-circuit measurements** for quantum computational tasks?

- Quantum random sampling is "hard" up to constant error in $||.||_{l_1}$ distance
- **Example:** Polynomial depth commuting IQP circuits with local gates



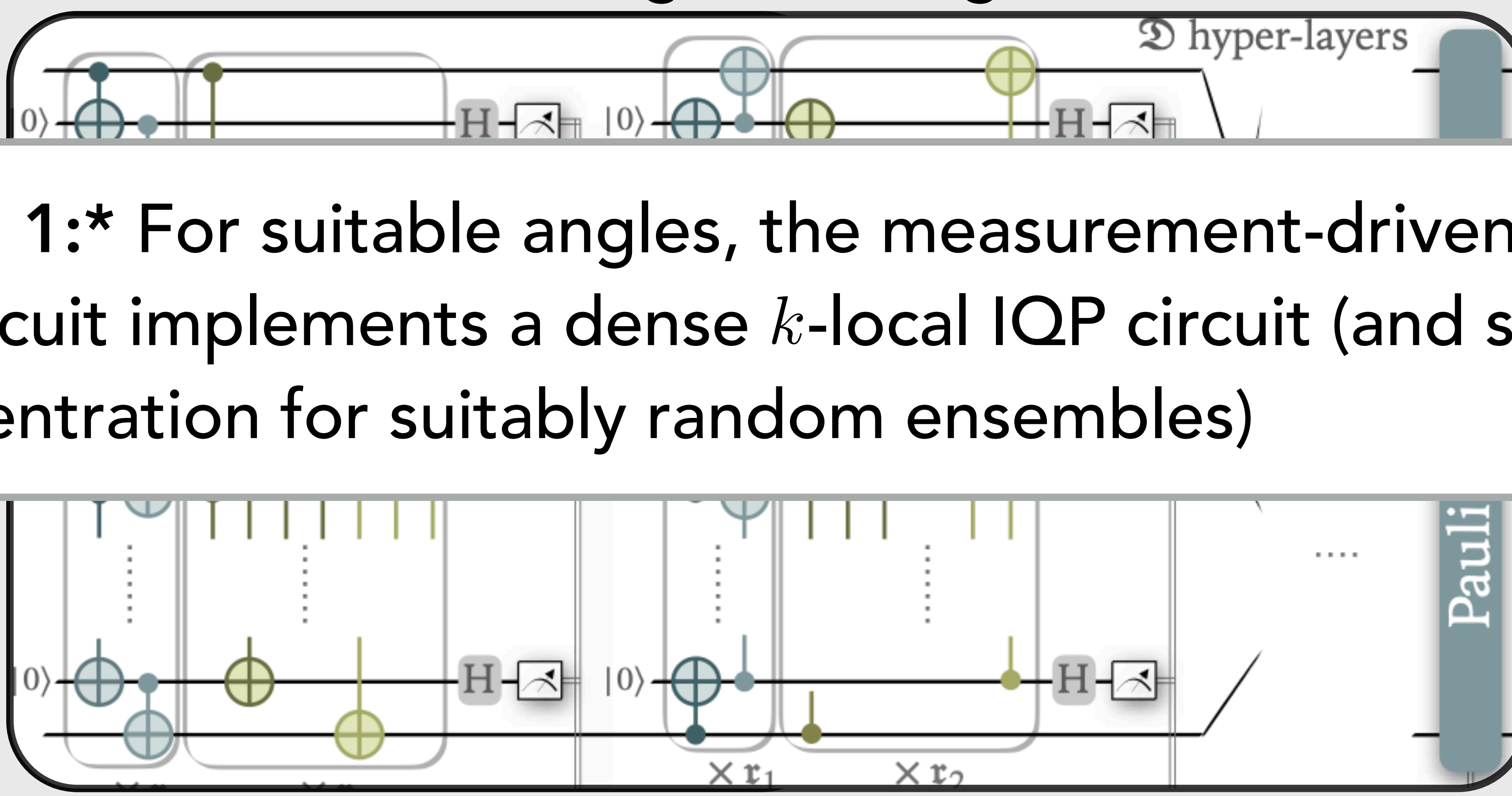
- Can measurements reduce the depth?

Shepherd, Bremner, Proc Roy Soc A 467, 459 (2011)
Hangleiter, Eisert, Rev Mod Phys 95, 035001 (2023)

MID-CIRCUIT MEASUREMENTS AS A RESOURCE

- How powerful are **mid-circuit measurements** for quantum computational tasks?

- Compressing dense IQP circuits to **constant depth** with mid-circuit measurements: Locally entangle system qubits with auxiliary qubits using a short pattern of nearest-neighbor CX gates - then measure X



- **Theorem 1:*** For suitable angles, the measurement-driven constant-depth circuit implements a dense k-local IQP circuit (and show anticoncentration for suitably random ensembles)

* All informal

Cao, Eisert, Phys Rev Lett 136, 080601 (2026)

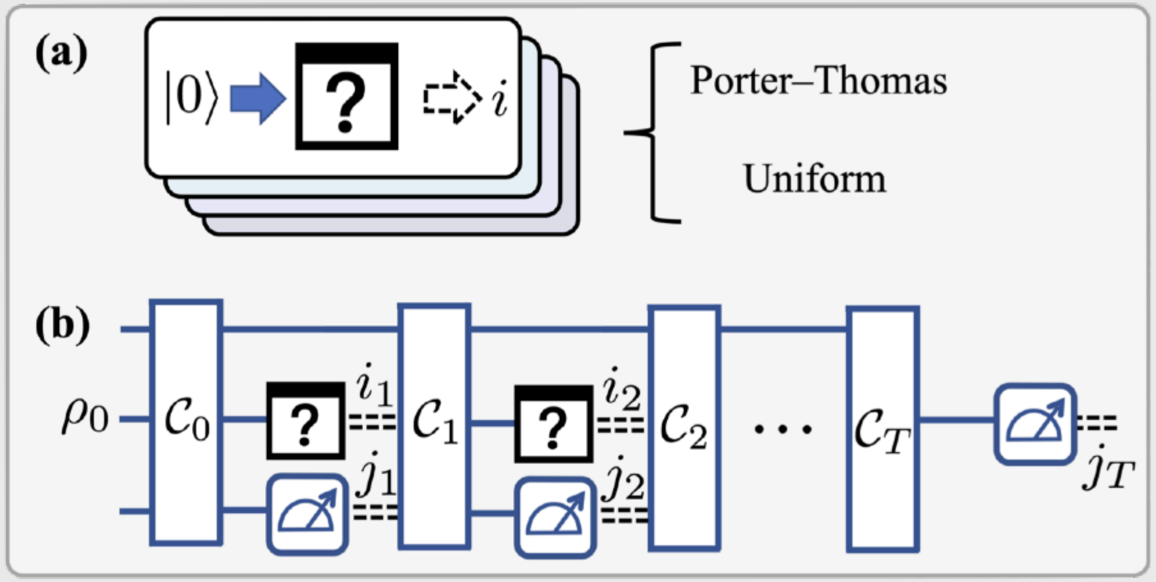
POST-MEASUREMENT STATES AS A RESOURCE

- How valuable are **post-measurements** states in estimating properties of measurements?

- Given query access to a measurement decide, the task is to distinguish between two i.i.d. occurring scenarios:
  - Decice outputs $i \in \{0, \ldots, d-1\}$ uniformly at random and leaves state unchanged
  - Device performs projective measurements

$$\{\Pi_i = U|i\rangle\langle i|U_i^\dagger\}_{i=0}^{d-1}$$

  with Haar random U



- **Theorem 2:** Needs $O(\sqrt{d})$ many measurements, even with adaptivity, auxiliary qubits, entangled operation, and controllable measurements

Liu, Ye, Cai, Eisert, arXiv:2602.22126 (2026)

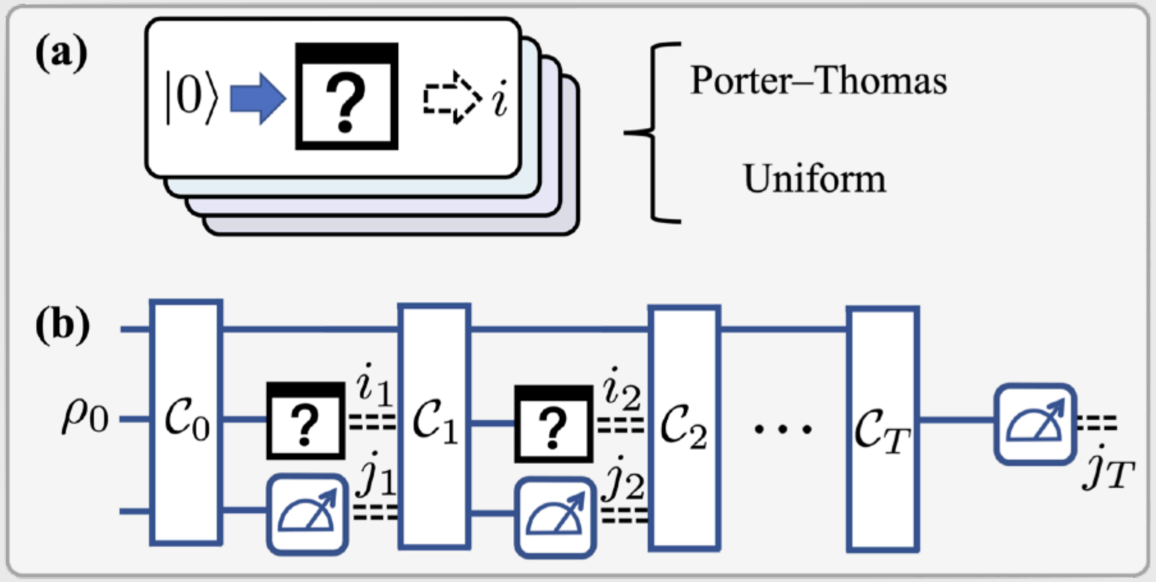
POST-MEASUREMENT STATES AS A RESOURCE

- How valuable are **post-measurements** states in estimating properties of measurements?

- Given query access to a measurement decide, the task is to distinguish between two i.i.d. occurring scenarios:
  - Decice outputs $i \in \{0, \ldots, d-1\}$ uniformly at random and leaves state unchanged
  - Device performs projective measurements

$$\{\Pi_i = U|i\rangle\langle i|U_i^\dagger\}_{i=0}^{d-1}$$

with Haar random U

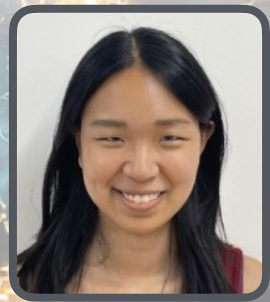
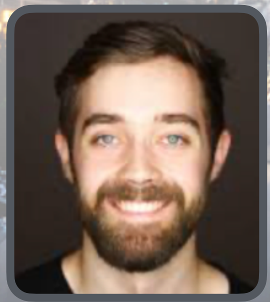


- **Theorem 2:** Needs $O(\sqrt{d})$ many measurements, even with adaptivity, auxiliary qubits, entangled operation, and controllable measurements - but with access to **post-measurement states** this is $O(1)$

Liu, Ye, Cai, Eisert, arXiv:2602.22126 (2026)

THE ROLE OF THE OBSERVER FOR RESOURCES

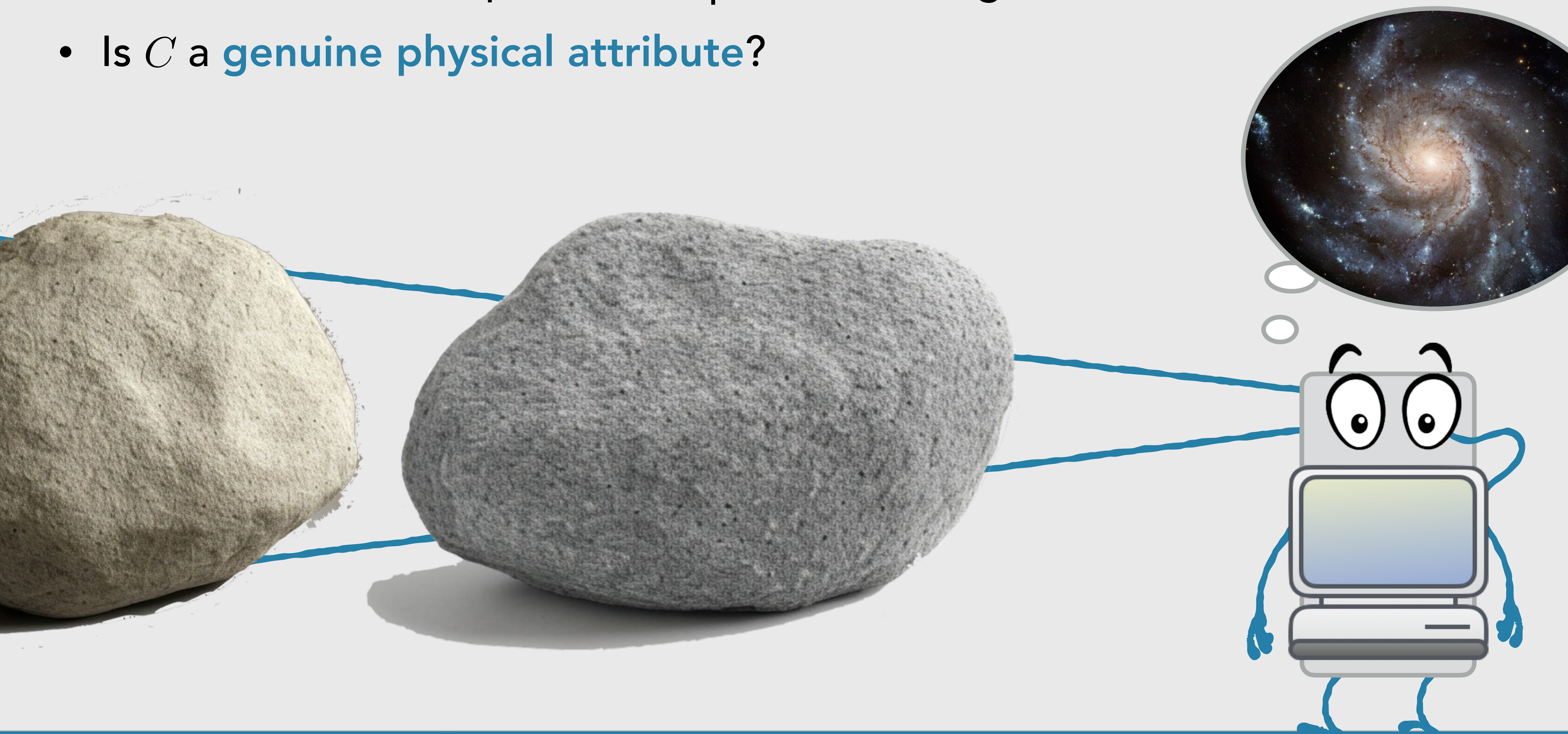
Leone, Rizzo, Eisert, Jerbi, Nature Phys 21, 1847 (2025)
Gu, Leone, Ghosh, Eisert, Yelin, Quek, Phys Rev Lett 132, 210602 (2024)
Meyer, Rizzo, Raza, Leone, Jerbi, Eisert, arXiv:2601.15393 (2026)
Meyer, Raza, Rizzo, Leone, Jerbi, Eisert, arXiv:2509.20472 (2025)
Gu, Quek, Yelin, Eisert, Leone, arXiv:2410.18196 (2024)

COMPUTATIONALLY-BOUNDED OBSERVERS

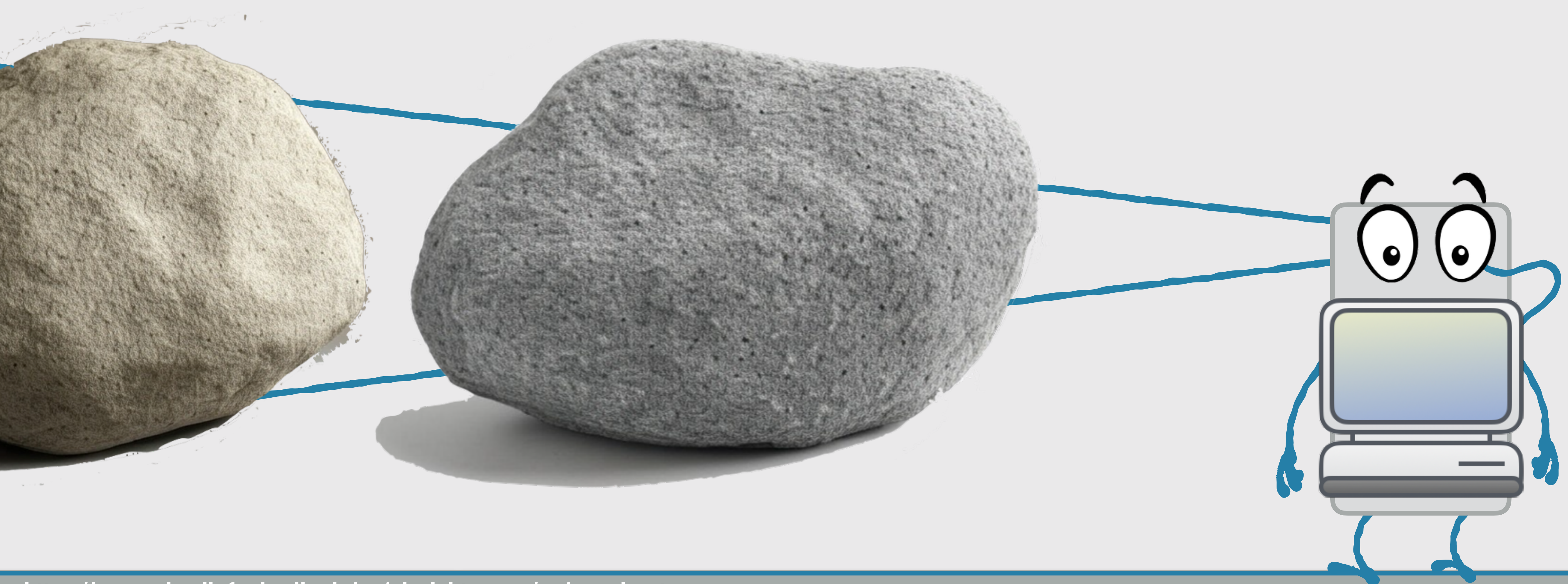
- Given two **quantum systems** that differ in a physical characteristic C, we aim to **distinguish** between them

- What if the **time** required is on par with the age of the universe?
- Is C a **genuine physical attribute**?



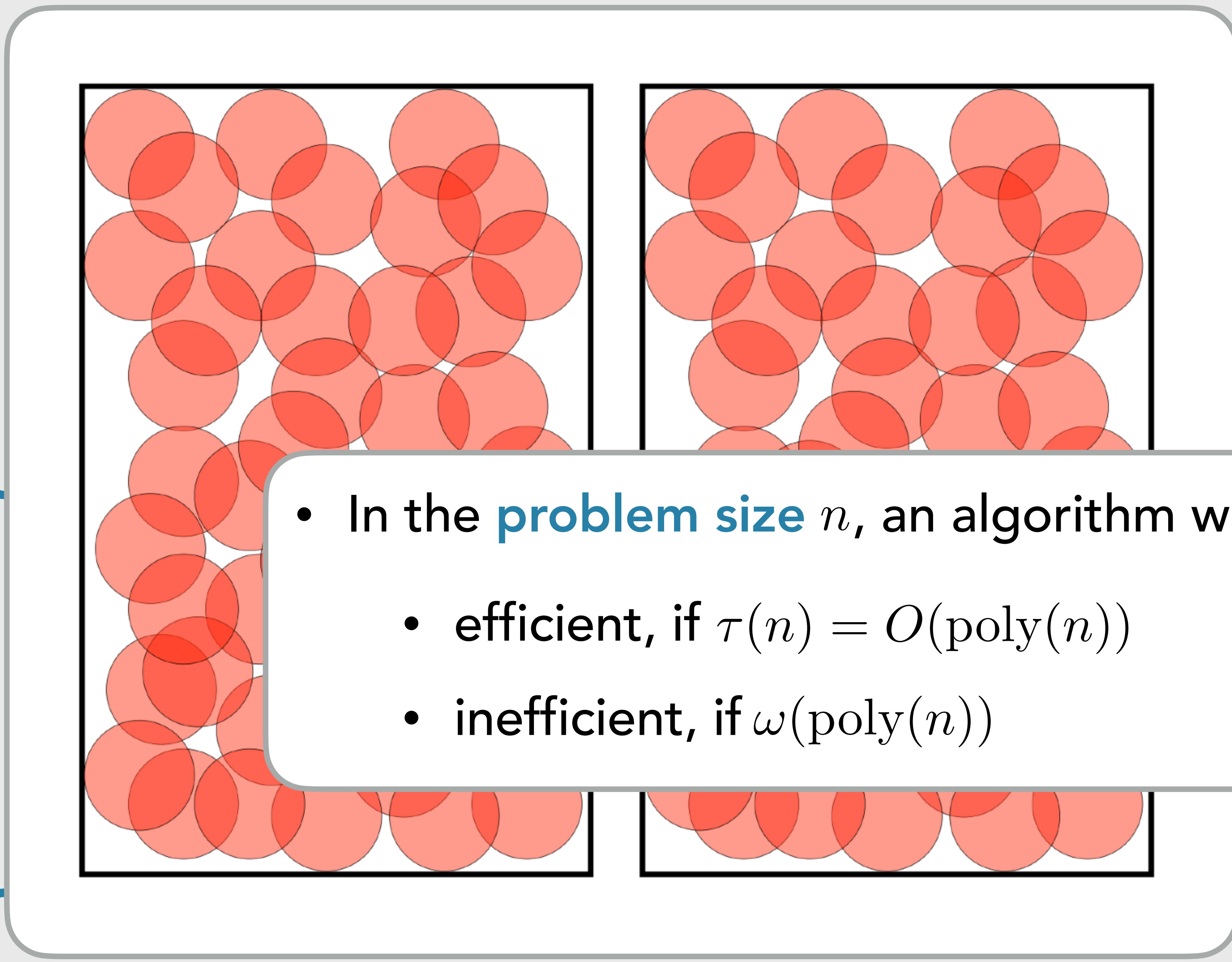
COMPUTATIONALLY-BOUNDED OBSERVERS

- **Definition** [*Computationally bounded observer*]: An observer taking measurements and operating for ‘a reasonable amount of time’.

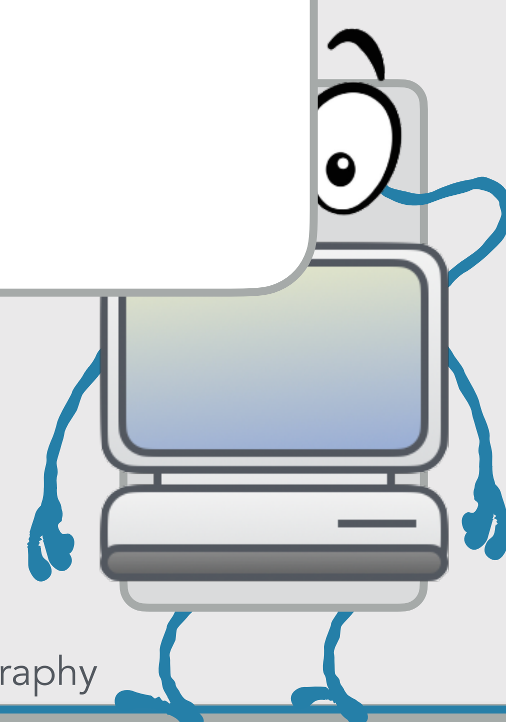


COMPUTATIONAL INDISTINGUISHABILITY

- Are the two boxes the **same or different**?



- In the **problem size** n, an algorithm with time $\tau(n)$ is
  - efficient, if $\tau(n) = O(\mathrm{poly}(n))$
  - inefficient, if $\omega(\mathrm{poly}(n))$

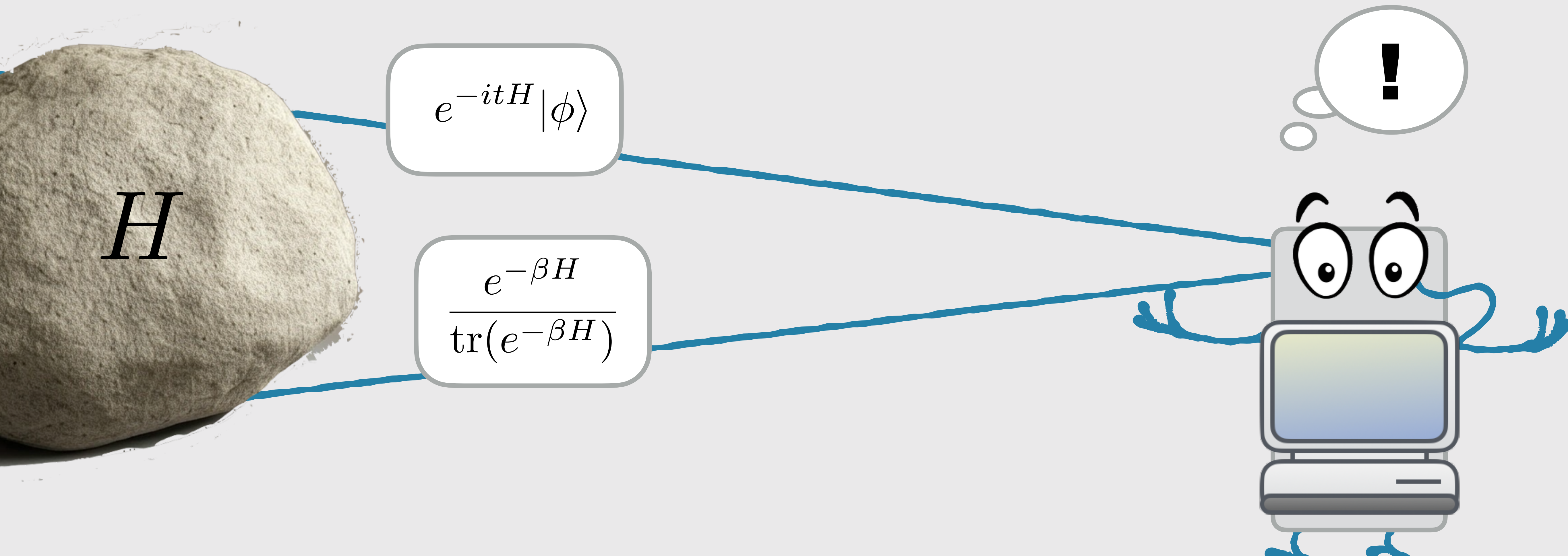


Pass and Shelat, A course in Cryptography

HAMILTONIAN ACCESS

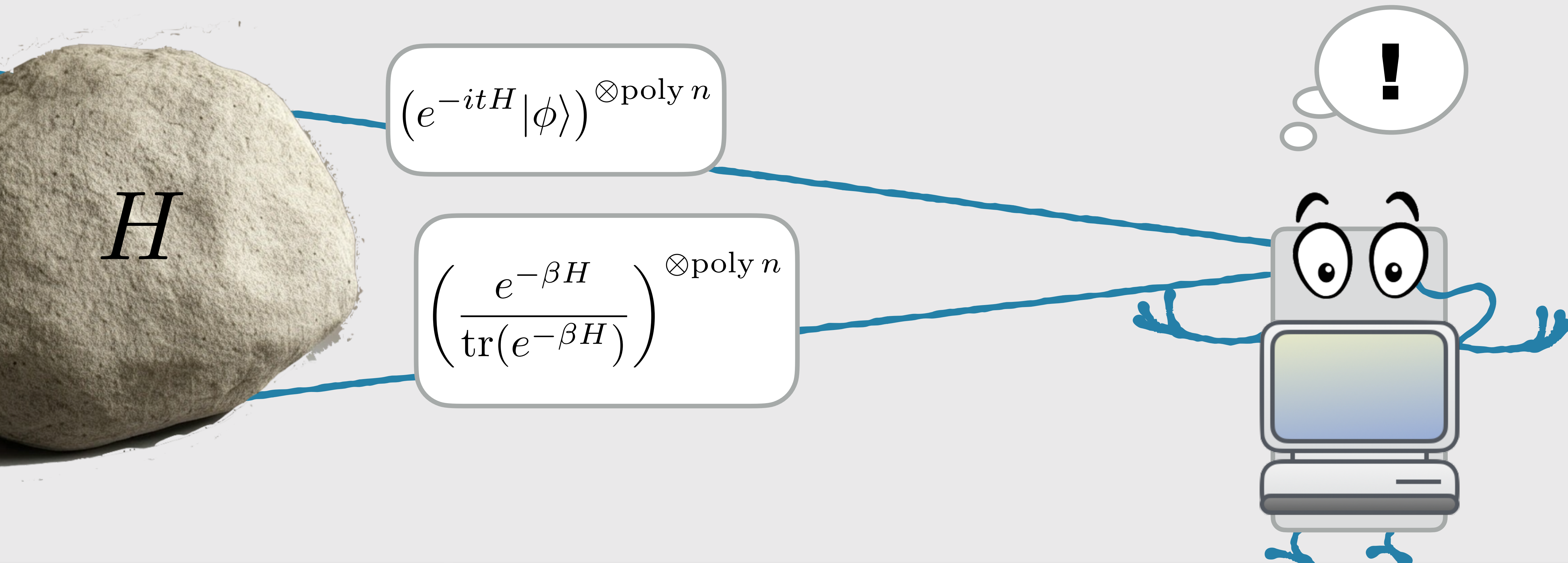
- What is reasonable access to an **unknown Hamiltonian**?

- **Definition** [*Hamiltonian access*]: An algorithm $\mathcal{A}^H$ has **black-box Hamiltonian access** if it can access to the time evolution e^{-itH} for **any** reasonable time $|t| = O(\mathrm{poly}(n))$, as well as has access to the Gibbs state $e^{-\beta H}$ for reasonably small temperatures $\beta = O(\mathrm{poly}(n))$



COMPUTATIONALLY INDISTINGUISHABLE SYSTEMS

- **Definition** [*Computationally indistinguishable physical systems*]: Two ensemble of Hamiltonians $\mathcal{E}_1 = \{H_j\}$ and $\mathcal{E}_2 = \{H_k\}$ are computationally indistinguishable if no efficient quantum algorithm can distinguish given black-box Hamiltonian access to samples from $\mathcal{E}_1$ and samples from $\mathcal{E}_2$



- What is **quantum chaos**?

- It's complicated
    - Naive butterfly effect does not work

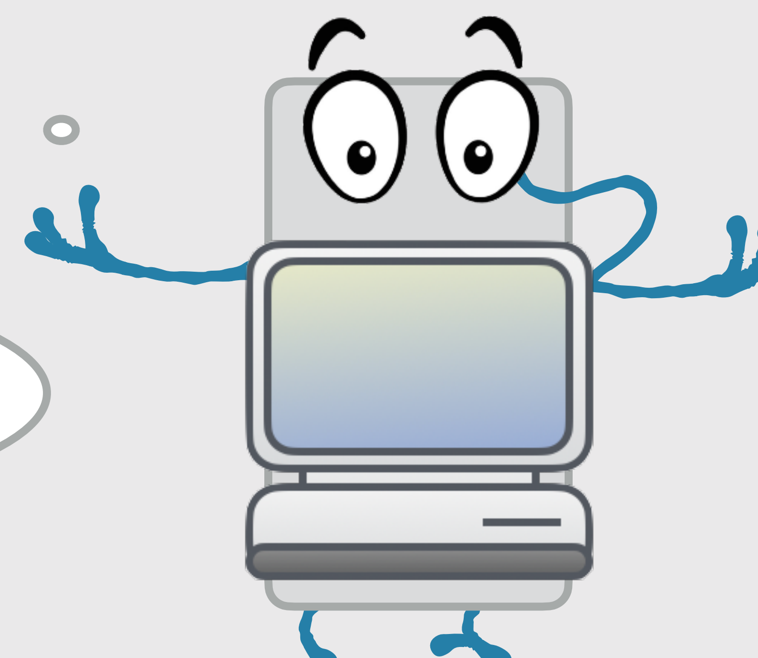
$$\langle\psi|\phi\rangle = \langle\psi|UU^\dagger|\phi\rangle$$

    - There are naturally 2^n constants of motion

$$H = \sum_i E_i |E_i\rangle\langle E_i| \Rightarrow [H, |E_i\rangle\langle E_i|] = 0$$

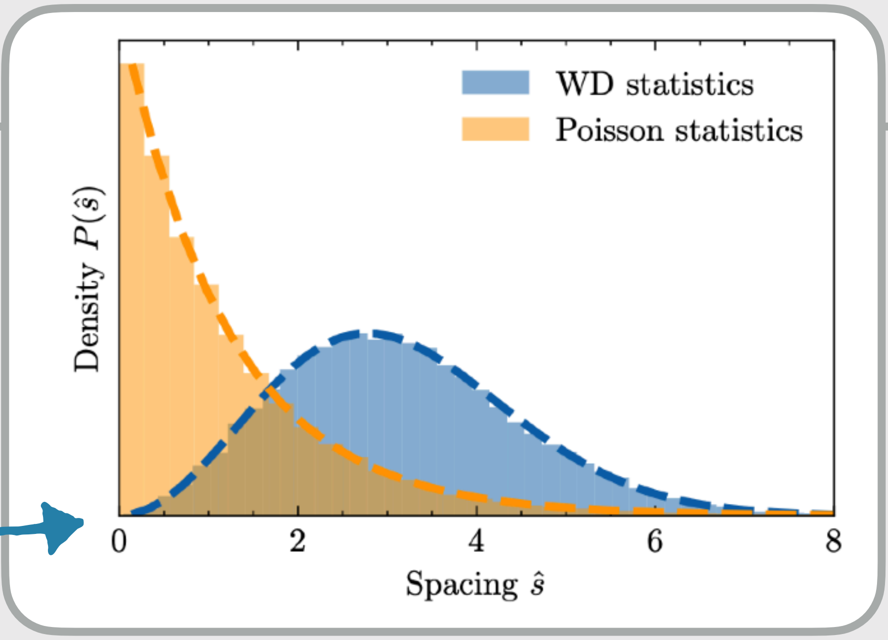
Quantum systems with level statistics resembling chaos

Semi-classical approaches



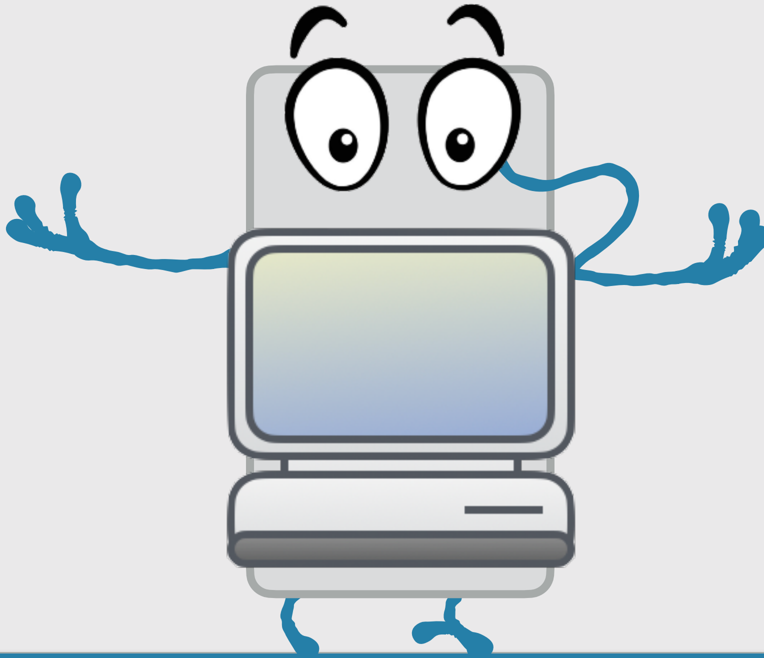
QUANTUM CHAOS

- Level spacing
  - Gaps $s_i = E_{i+1} - E_i$



Level repulsion

- Wigner-Dyson stat associated with quantum chaos
- Poisson statistics associated with integrability*

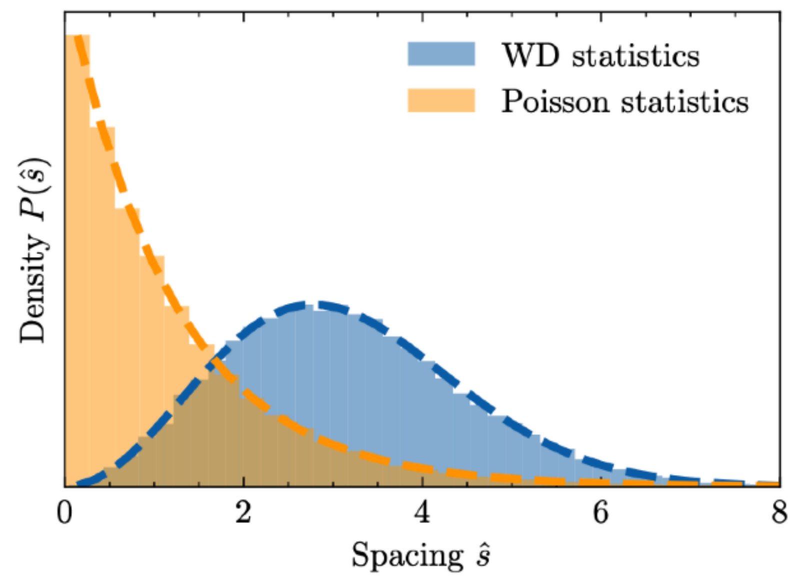


* Careful, exist integrable Wigner-Dyson distributed quantum systems

Benet, Leyvraz, Seligman, Phys Rev E 68, 045201(R) (2003)

QUANTUM CHAOS

- **Level spacing**
  - Gaps $s_i = E_{i+1} - E_i$



- Localised quantum info in local operator O_1 spreads by e^{-iHt}: **quantum information scrambling** $e^{-iHt}O_1e^{iHt}$

  - Scrambling measured by a test operator O_2 via the decay of the "**out-of-time-ordered-correlation functions**" (OTOCs)

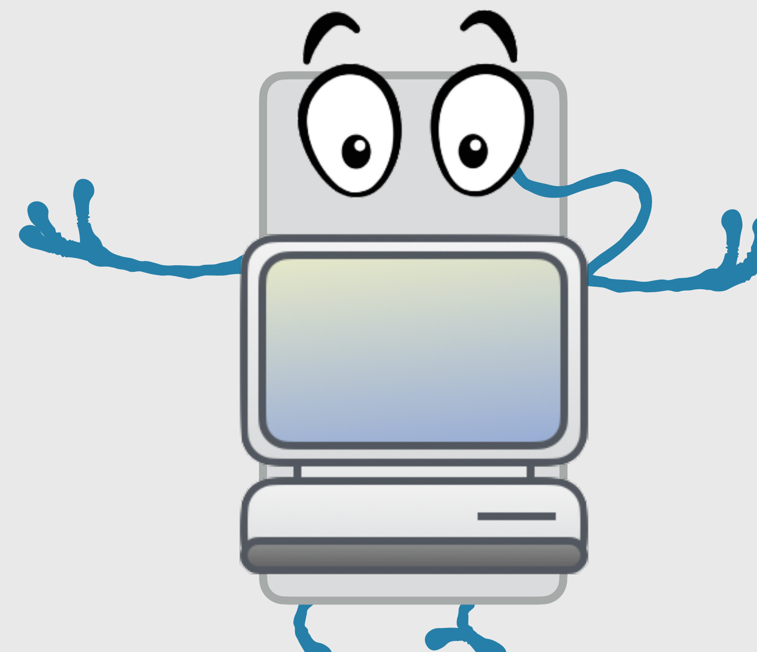
$$\mathrm{OTOC}(H,T) = \langle O_1(t)O_2O_1(t)O_2\rangle = O(\exp(-n))$$

  - Scrambling measured by the **operator entanglement**

$$\mathrm{LOE}(H,T) = S_{A|B}(|O_1(t)\rangle) = \Omega(n)$$

for Choi state $|O_1(t)\rangle = (O_1(t)\otimes I)|\Omega\rangle$,

$$|\Omega\rangle = \frac{1}{2^{n/2}}\sum_{j=1}^{d}|j,j\rangle$$



GAUSSIAN UNITARY ENSEMBLE

- **Definition [*GUE*]:** $\mathcal{E}_1 = \{H_j\}$
  - Ensemble of Hamiltonians with unitarily invariant measure

$$dH = e^{-\frac{d}{2}\mathrm{tr}(H^2)}, \ d = 2^n$$

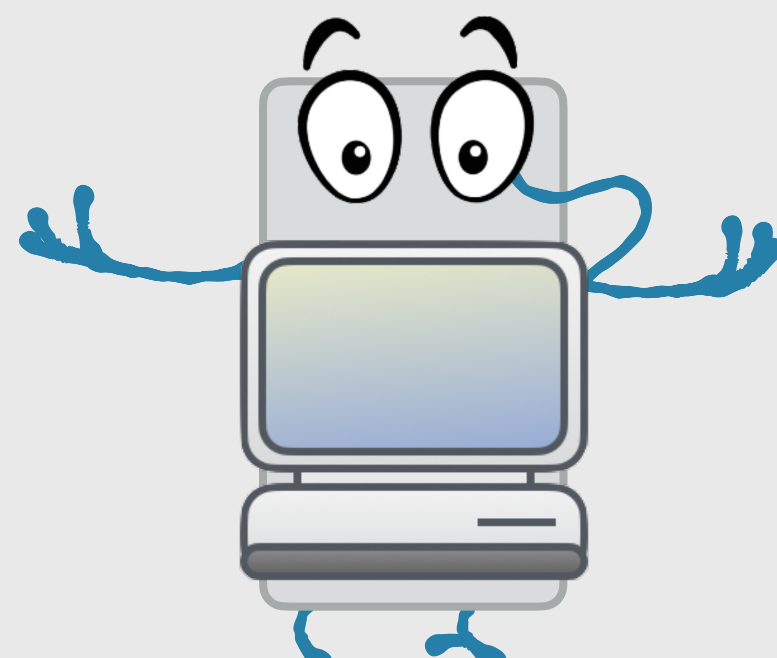
  - Characterized by Hamiltonians

$$H = \sum_i E_i |\psi_i\rangle\langle\psi_i|$$

with $|\psi_i\rangle \sim$ Haar measure and

$$\lambda_i \sim p(\lambda_1, \ldots, \lambda_d) \sim e^{-\frac{d}{2}(\lambda_1 + \cdots + \lambda_d)^2} \prod_{i>j} |\lambda_i - \lambda_j|^2$$

Level repulsion



GAUSSIAN UNITARY ENSEMBLE

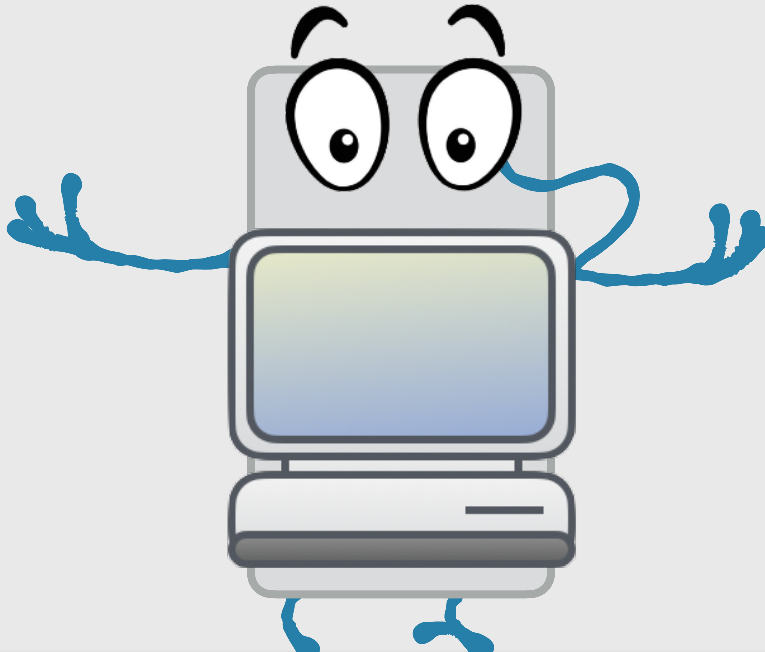
- **Definition [*GUE*]:** $\mathcal{E}_1 = \{H_j\}$
  - Level repulsion
  - Deep scrambling

    $$\mathrm{OTOC}_4(H,t) < \frac{1}{\omega(\exp(n))}$$

  - Highly entangled states

    $$S_2(H,t) > \alpha n$$

- The GUE paradigmatically fulfills those **chaotic properties**



GAUSSIAN UNITARY ENSEMBLE

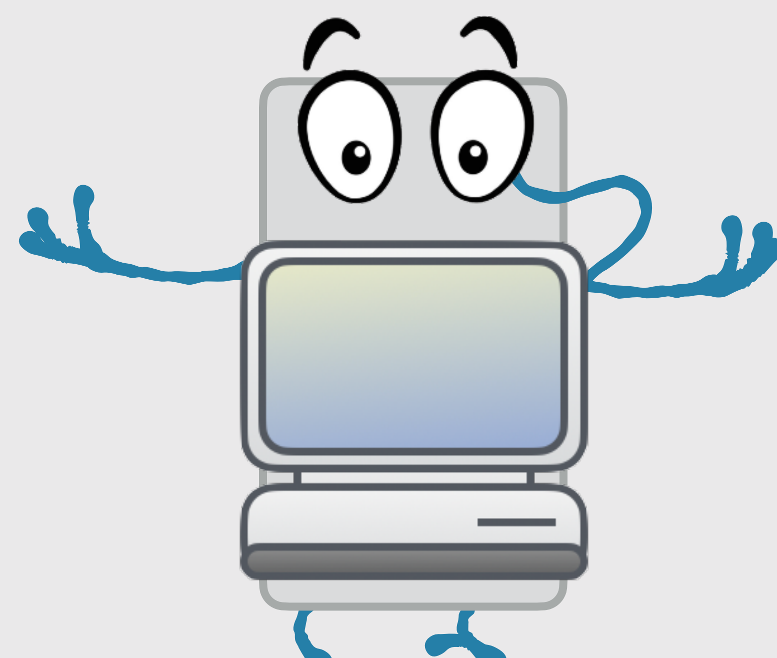
- **Definition [*GUE*]:** $\mathcal{E}_1 = \{H_j\}$
  - Level repulsion
  - Deep scrambling

    $$\mathrm{OTOC}_4(H,t) < \frac{1}{\omega(\exp(n))}$$

  - Highly entangled states

    $$S_2(H,t) > \alpha n$$

- **A new ensemble** $\mathcal{E}_2 = \{H_k\}$
  - It can be efficiently quantum simulated



GAUSSIAN UNITARY ENSEMBLE

- **Definition [*GUE*]:** $\mathcal{E}_1 = \{H_j\}$
  - Level repulsion
  - Deep scrambling

    $$\mathrm{OTOC}_4(H,t) < \frac{1}{\omega(\exp(n))}$$

  - Highly entangled states

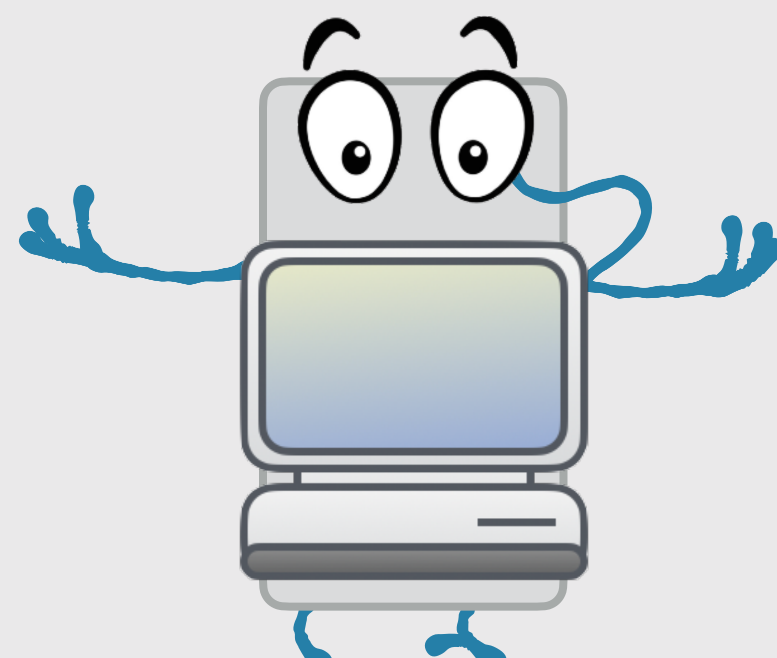
    $$S_2(H,t) > \alpha n$$

- **A new ensemble** $\mathcal{E}_2 = \{H_k\}$
  - No level repulsion
  - OTOC saturating at levels

    $$\mathrm{OTOC}_4(H,t) \geq \frac{1}{\omega(\mathrm{poly}(n))}$$

  - Weakly entangled states

    $$S_2(H,t) \leq \mathrm{poly}\log(n)$$



GAUSSIAN UNITARY ENSEMBLE

- **Definition [*GUE*]:** $\mathcal{E}_1 = \{H_j\}$
  - Level repulsion
  - Deep scrambling

    $$\mathrm{OTOC}_4(H,t) < \frac{1}{\omega(\exp(n))}$$
  - Highly entangled states

    $$S_2(H,t) > \alpha n$$

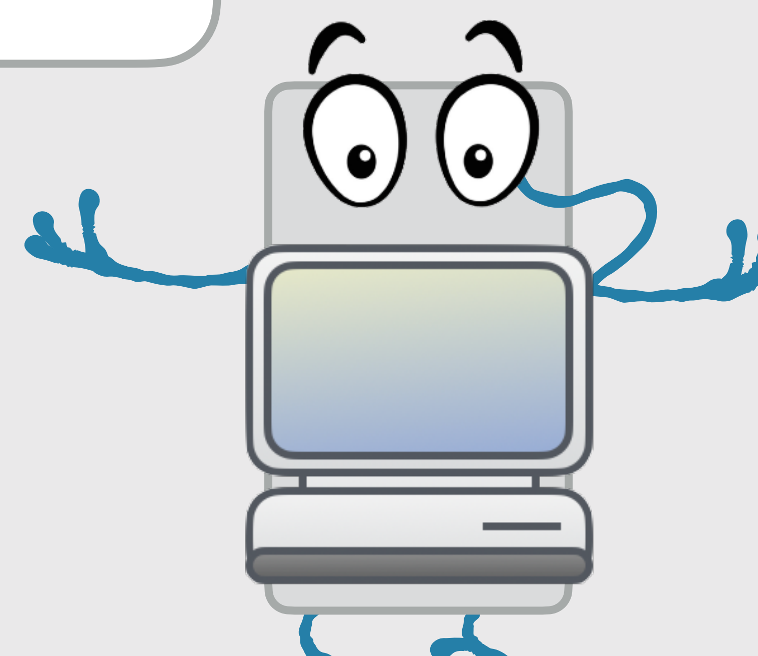
- **A new ensemble** $\mathcal{E}_2 = \{H_k\}$
  - No level repulsion
  - OTOC saturating at levels

    $$\mathrm{OTOC}_4(H,t) \geq \frac{1}{\omega(\mathrm{poly}(n))}$$
  - Weakly entangled states

    $$S_2(H,t) \leq \mathrm{poly}\log(n)$$

- **Theorem 3 [*Pseudochaotic Hamiltonians*]:** [...]

  (i) it does **not** exhibit chaotic properties



GAUSSIAN UNITARY ENSEMBLE

- **Definition [*GUE*]:** $\mathcal{E}_1 = \{H_j\}$
  - Level repulsion
  - Deep scrambling

    $$\mathrm{OTOC}_4(H,t) < \frac{1}{\omega(\exp(n))}$$

  - Highly entangled states

    $$S_2(H,t) > \alpha n$$

- **A new ensemble** $\mathcal{E}_2 = \{H_k\}$
  - No level repulsion
  - OTOC saturating at levels

    $$\mathrm{OTOC}_4(H,t) \geq \frac{1}{\omega(\mathrm{poly}(n))}$$

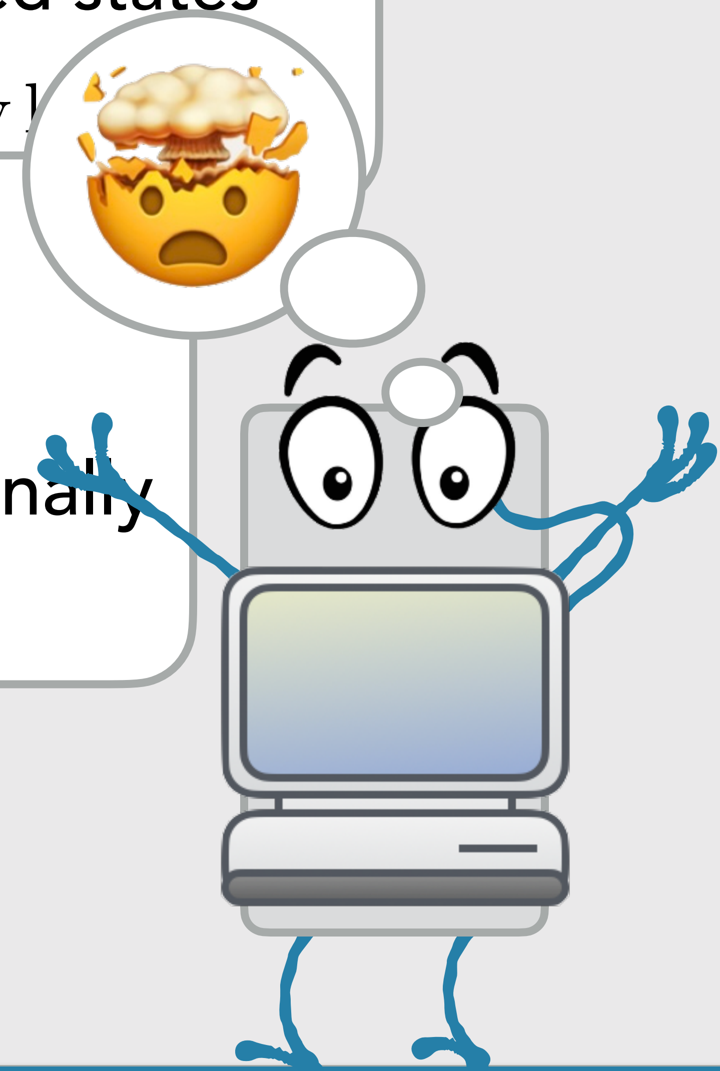
  - Weakly entangled states

    $$S_2(H,t) \leq \mathrm{poly}$$

- **Theorem 3 [*Pseudochaotic Hamiltonians*]:** […]

  (i) it does **not** exhibit chaotic properties

  (ii) It is **indistinguishable** from $\mathcal{E}_1$ to every computationally efficient algorithm with Hamiltonian access*



*Implies sample complexity bound for spectrum testing

GAUSSIAN UNITARY ENSEMBLE

- **Definition [*GUE*]:** $\mathcal{E}_1 = \{H_j\}$
  - Level repulsion
  - Deep scrambling

    $$\mathrm{OTOC}_4(H,t) < \frac{1}{\omega(\exp(n))}$$

  - Highly entangled states

    $S_2(H$ …

- **A new ensemble** $\mathcal{E}_2 = \{H_k\}$
  - No level repulsion
  - OTOC saturating at levels

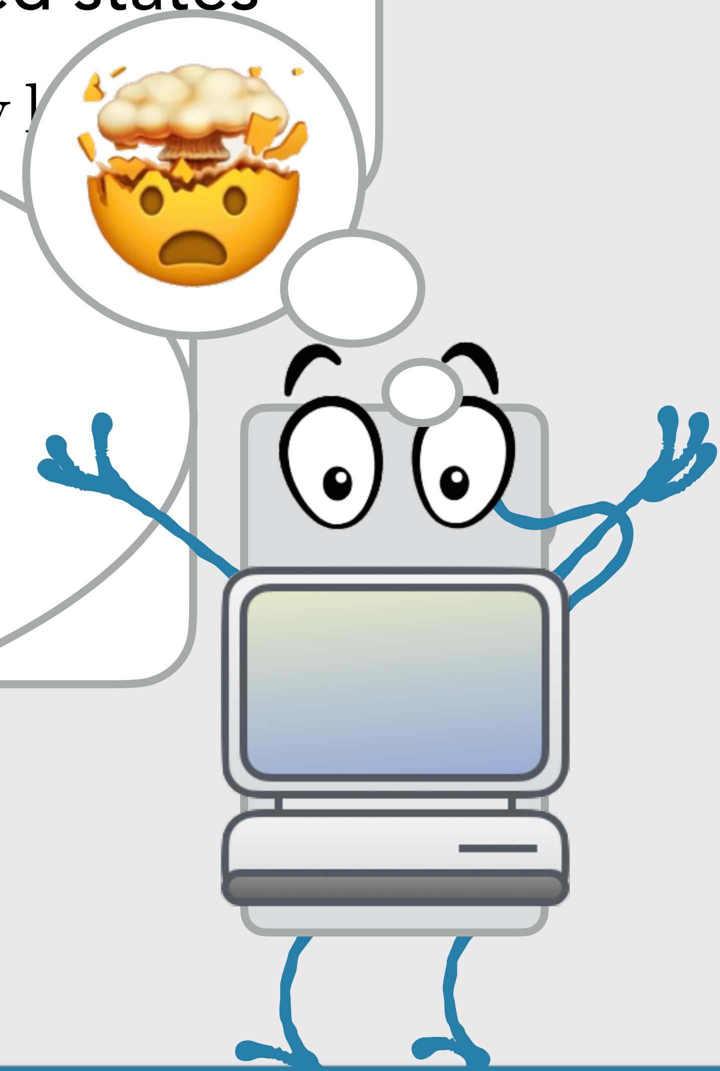
    $$\mathrm{OTOC}_4(H,t) \geq \frac{1}{\omega(\mathrm{poly}(n))}$$

  - Weakly entangled states

    … $t) \leq \mathrm{poly}$ …

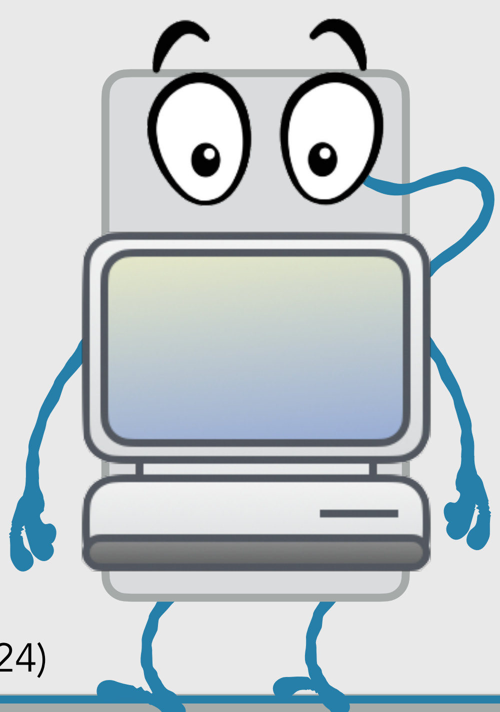
- **Th** …

- **Wait, what, what what?** It cannot exist!
- If it existed, all so-called 'necessary probes' of quantum chaos lead to the same emergent phenomena!



PSEUDOCHAOTIC HAMILTONIANS

- It does exist



Gu, Quek, Yelin, Eisert, Leone, arXiv:2410.18196 (2024)

PSEUDOCHAOTIC HAMILTONIANS

- "Spoof" eigenvalues and eigenvectors separately

$|\psi_i\rangle \sim$ pseudorandom state

$$\lambda_i \sim \tilde{p}(\lambda_1, \ldots, \lambda_{\tilde{d}}) = \prod_{j=1}^{\tilde{d}} p^{(1)}(\lambda_j)$$

I.i.d. distribution

$$p^{(1)}(\lambda) = \frac{1}{4\pi}\sqrt{4 - \lambda^2}$$

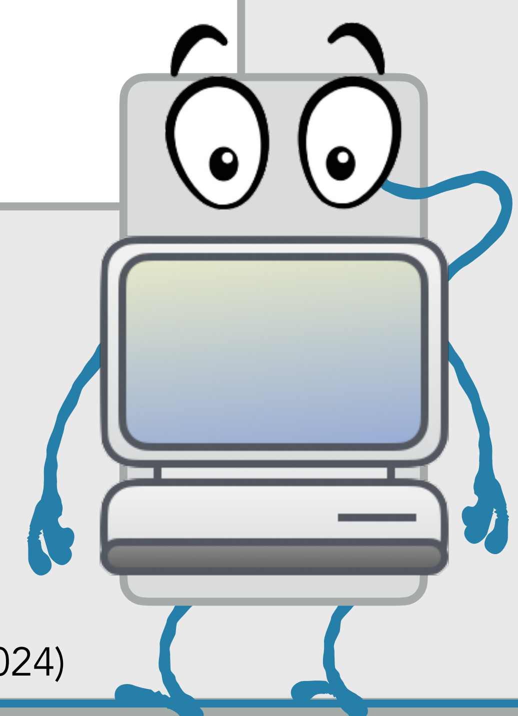
Wigner semi-circle law

$$\tilde{d} \ll d$$

Highly degenerate distribution

Independent energy levels - no repulsion

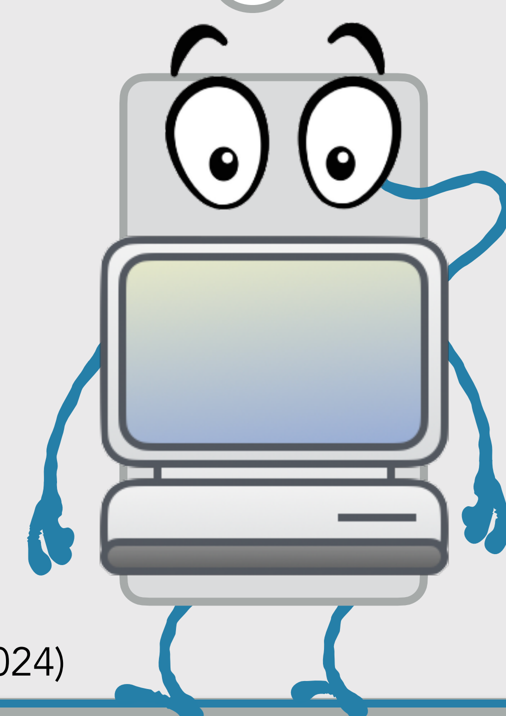
$$\mathcal{E}_d = \{\sum_i \lambda_i |\psi_i\rangle\langle i| : |\psi_i\rangle \in \text{pseudorandom}, \lambda \sim \tilde{p}\}$$



Gu, Quek, Yelin, Eisert, Leone, arXiv:2410.18196 (2024)

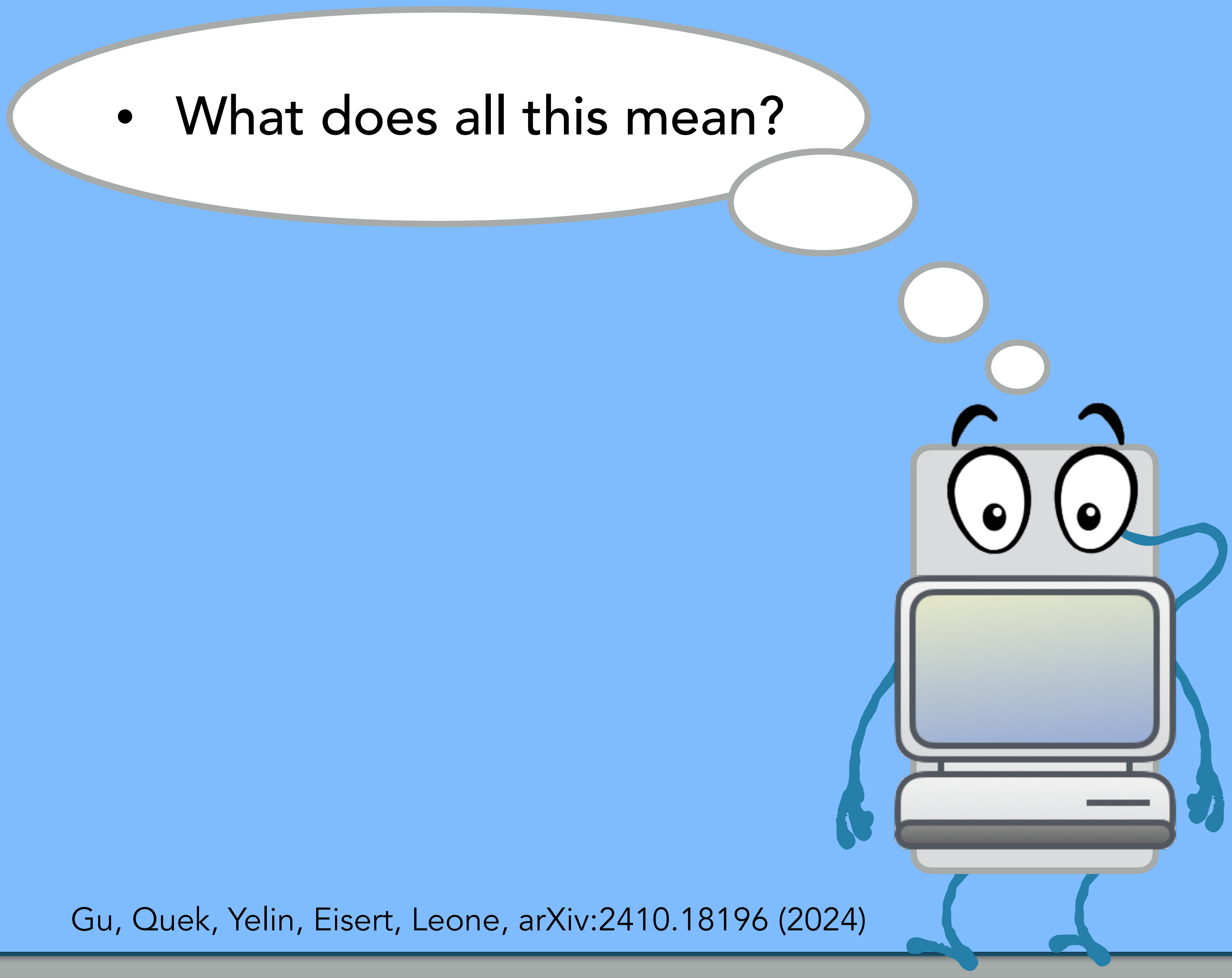
INTERPRETATION?

- What does all this mean?



Gu, Quek, Yelin, Eisert, Leone, arXiv:2410.18196 (2024)

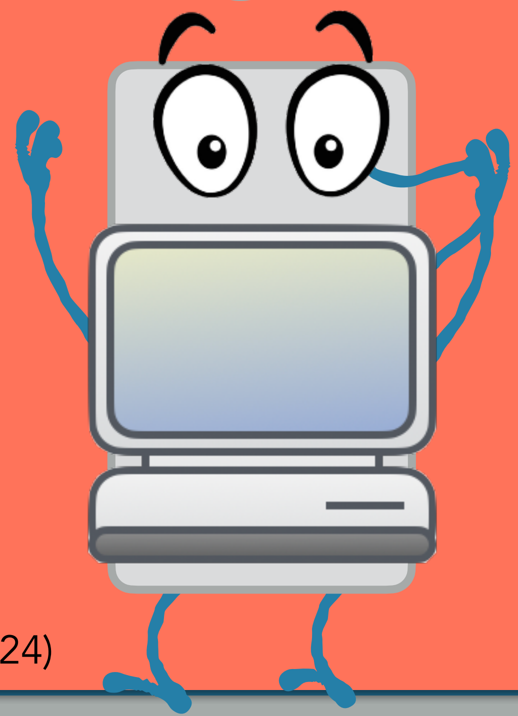
INTERPRETATION?



Gu, Quek, Yelin, Eisert, Leone, arXiv:2410.18196 (2024)

INTERPRETATION?

- What does all this mean?



Gu, Quek, Yelin, Eisert, Leone, arXiv:2410.18196 (2024)

A PROGRAMME IN RESOURCE THEORY

- **Computationally bounded devices** make a big difference in QIT

- **A programme:** Hypothesis testing and Stein's lemma under computational constraints

$$D_h^{\epsilon}(\rho \,\|\, \sigma) := -\log \min_{0 \leq \Lambda \leq \mathbb{I}} \left\{ \mathrm{Tr}[\Lambda\sigma] \,|\, \mathrm{Tr}[\Lambda\rho] \geq 1 - \epsilon \right\}$$

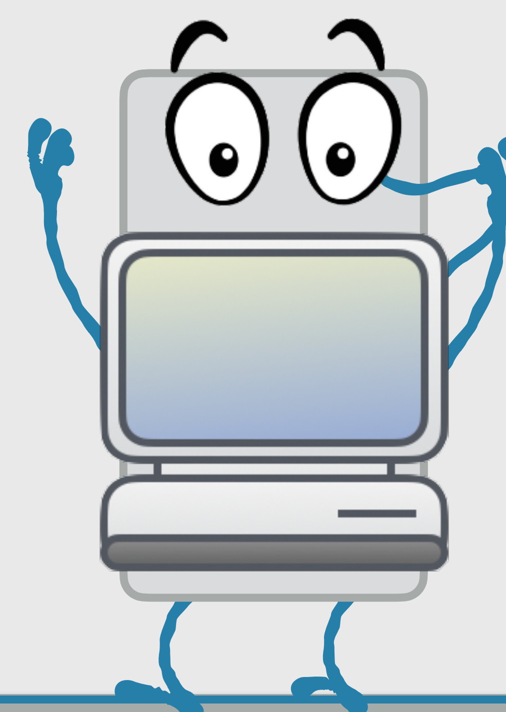
Meyer, Raza, Rizzo, Leone, Jerbi, Eisert, arXiv:2509.20472 (2025)

- There are channels with positive quantum capacity, but **zero computational capacity**

Meyer, Rizzo, Raza, Leone, Jerbi, Eisert, arXiv:2601.15393 (2026)

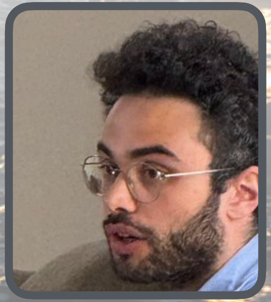
- There exists **pseudomagic**

Gu, Leone, Ghosh, Eisert, Yelin, Quek, Phys Rev Lett 132, 210602 (2024)



DECIDING ABOUT MAGIC

Leone, Oliviero, arXiv:2602.22330 (2026)



- Magic is a resource responsible for quantum speedups

Potential quantum advantages

- Where is the fine line?

Efficient classical simulation with Pauli propagation methods

- Does a state contain magic?

WEAK MEMBERSHIP PROBLEM FOR THE STABILIZER POLYTOPE

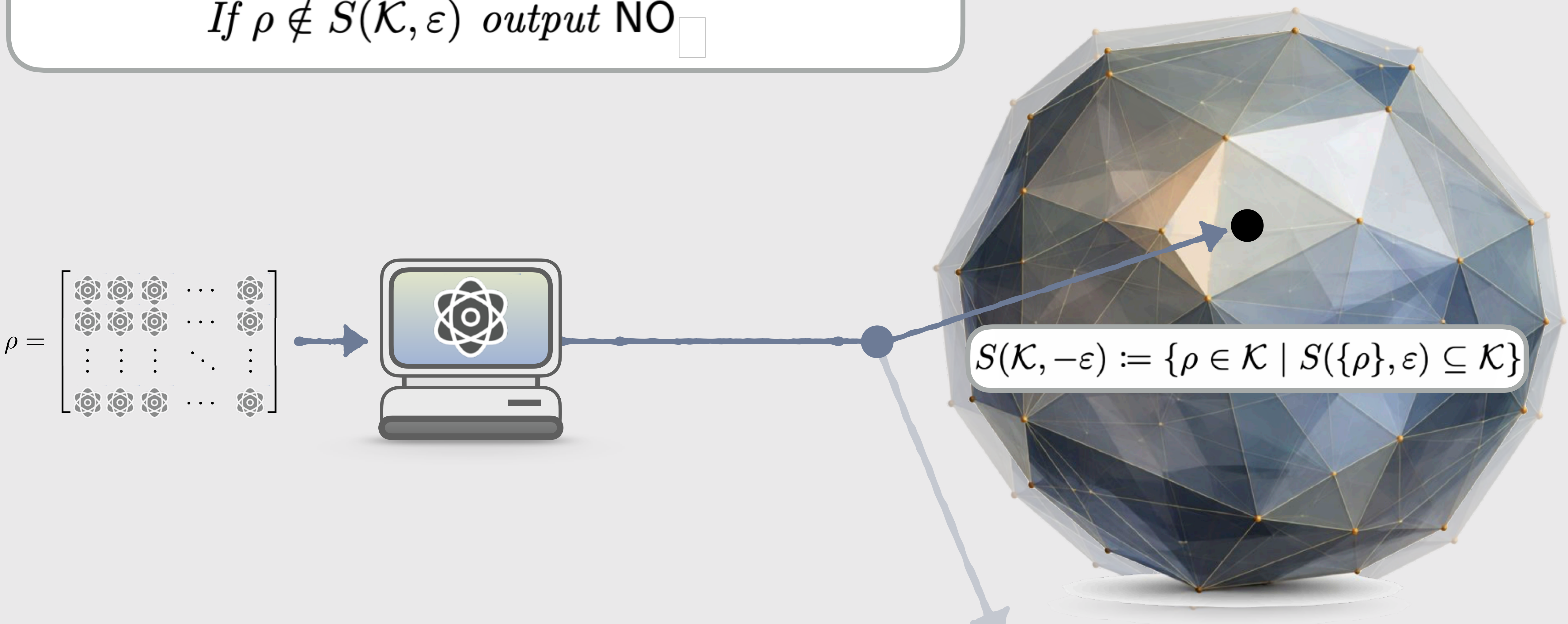
- Does a state contain **magic**?

WEAK MEMBERSHIP PROBLEM FOR THE STABILIZER POLYTOPE

- Weak membership problem of a state ρ being contained in the stabilizer polytope

$$\textit{If } \rho \in S(\mathcal{K}, -\varepsilon) \ \textit{ output } \text{YES}$$

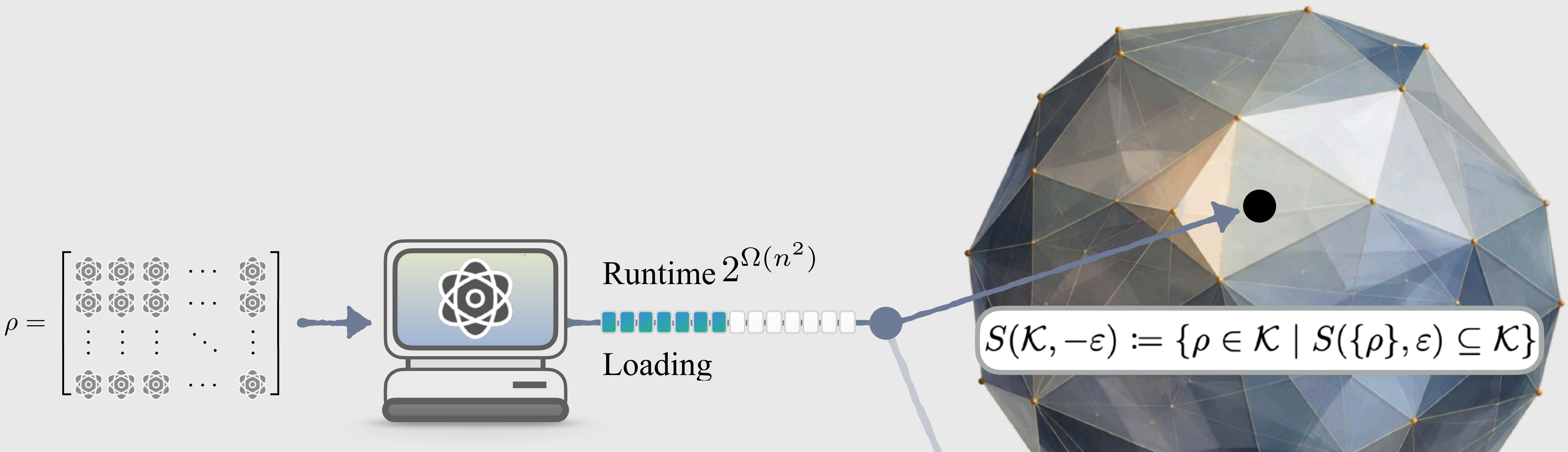
$$\textit{If } \rho \notin S(\mathcal{K}, \varepsilon) \ \textit{ output } \text{NO}$$



$$S(\mathcal{K}, -\varepsilon) := \{\rho \in \mathcal{K} \mid S(\{\rho\}, \varepsilon) \subseteq \mathcal{K}\}$$

$$S(\mathcal{K}, \varepsilon) := \{\rho \in \mathcal{H}_n \mid \exists \sigma \in \mathcal{K} : \|\rho - \sigma\|_2 \leq \varepsilon\}$$

- **Theorem 4**: Deciding whether a state ρ lies in the stabilizer polytope $\hat{S}$, or is ε-far from every state in $\hat{S}$, belongs to the complexity class $\mathtt{QP}^2$ for any $\varepsilon = 1/\mathrm{poly}(d)$ *



* For $k \in \mathbb{R}_+$, a decision problem with input size d is said to lie in $\mathtt{QP}^k$ if there exists a deterministic Turing machine solving it in time $\exp(\log^k d)$

$$S(\mathcal{K}, \varepsilon) := \{\rho \in \mathcal{H}_n \mid \exists \sigma \in \mathcal{K} : \|\rho - \sigma\|_2 \leq \varepsilon\}$$

Leone, Oliviero, arXiv:2602.22330 (2026)

A GLIMPSE AT THE PROOF

- **Theorem 4**: Deciding whether a state ρ lies in the stabilizer polytope $\hat{S}$, or is ε-far from every state in $\hat{S}$, belongs to the complexity class $\texttt{QP}^2$ for any $\varepsilon = 1/\mathrm{poly}(d)$

The decision problem lies in $\texttt{QP}^2$

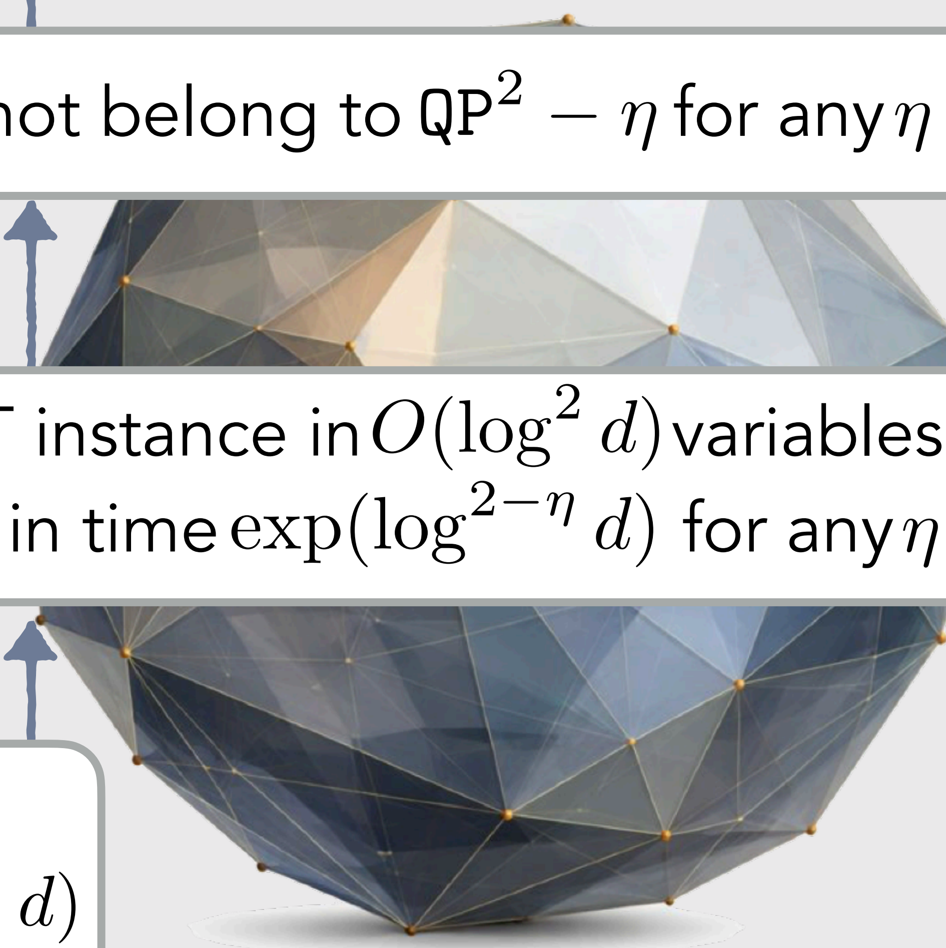
Compute robustness of magic, a magic **monotone**, in time $\exp(\log^2 d)$

$$\mathcal{R}(\rho) := \min_x \left\{ \sum_i |x_i| \;:\; \rho = \sum_i x_i \sigma_i,\ \sigma_i \in \mathcal{S}_n \right\}$$

Leone, Oliviero, arXiv:2602.22330 (2026)
Howard, Campbell, Phys Rev Lett 118, 090501 (2017)

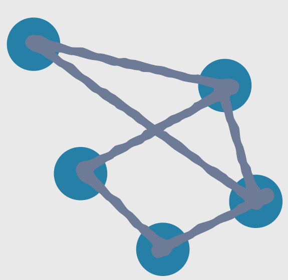
A GLIMPSE AT THE PROOF

- **Theorem 4**: Deciding whether a state ρ lies in the stabilizer polytope $\hat{S}$, or is ε-far from every state in $\hat{S}$, belongs to the complexity class $\mathtt{QP}^2$ for any $\varepsilon = 1/\mathrm{poly}(d)$



It does not belong to $\mathtt{QP}^2 - \eta$ for any $\eta > 0$

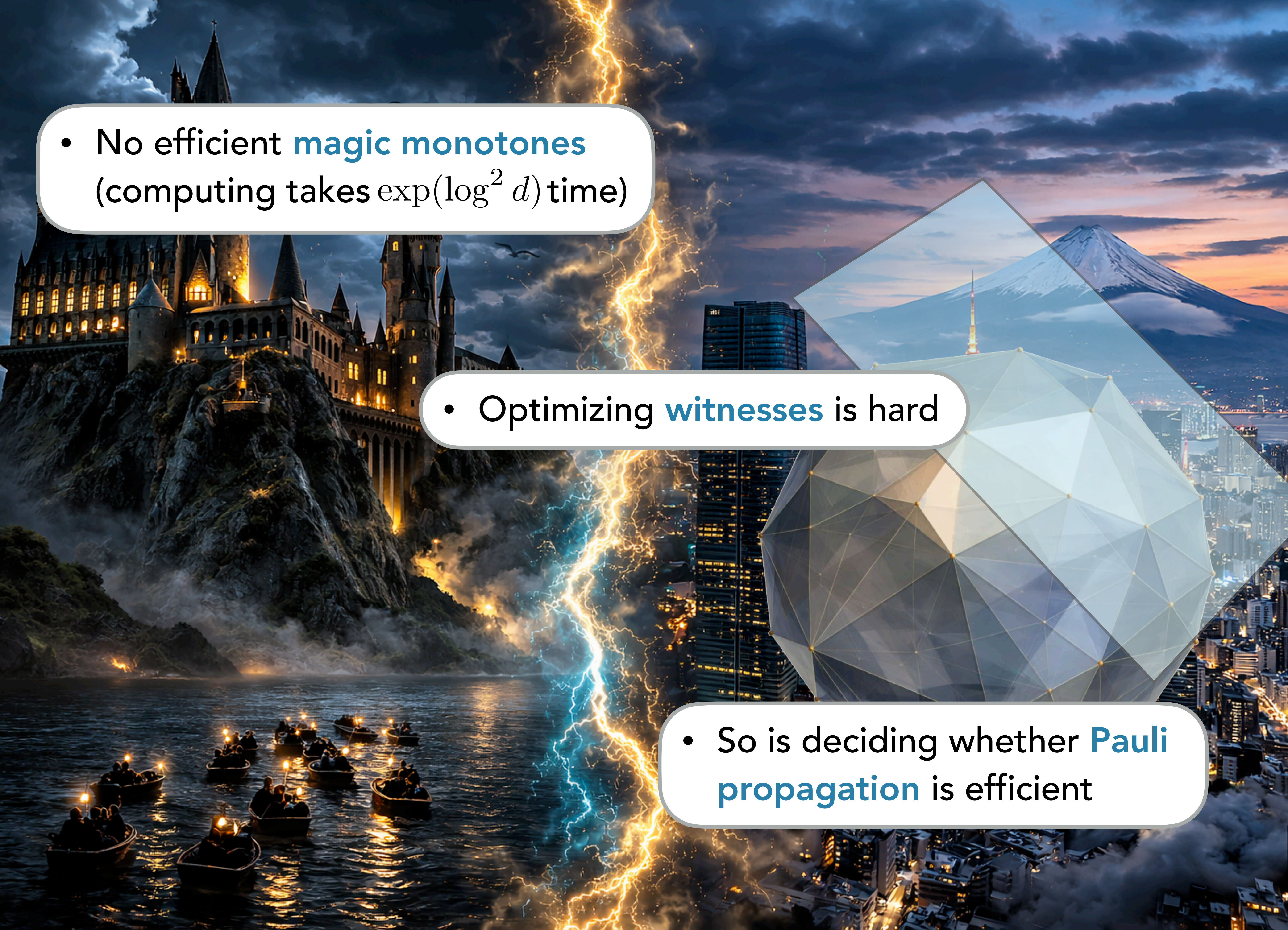
Encodes an instance as a 3-SAT instance in $O(\log^2 d)$ variables, by the ETH, this cannot be solved in time $\exp(\log^{2-\eta} d)$ for any $\eta > 0$



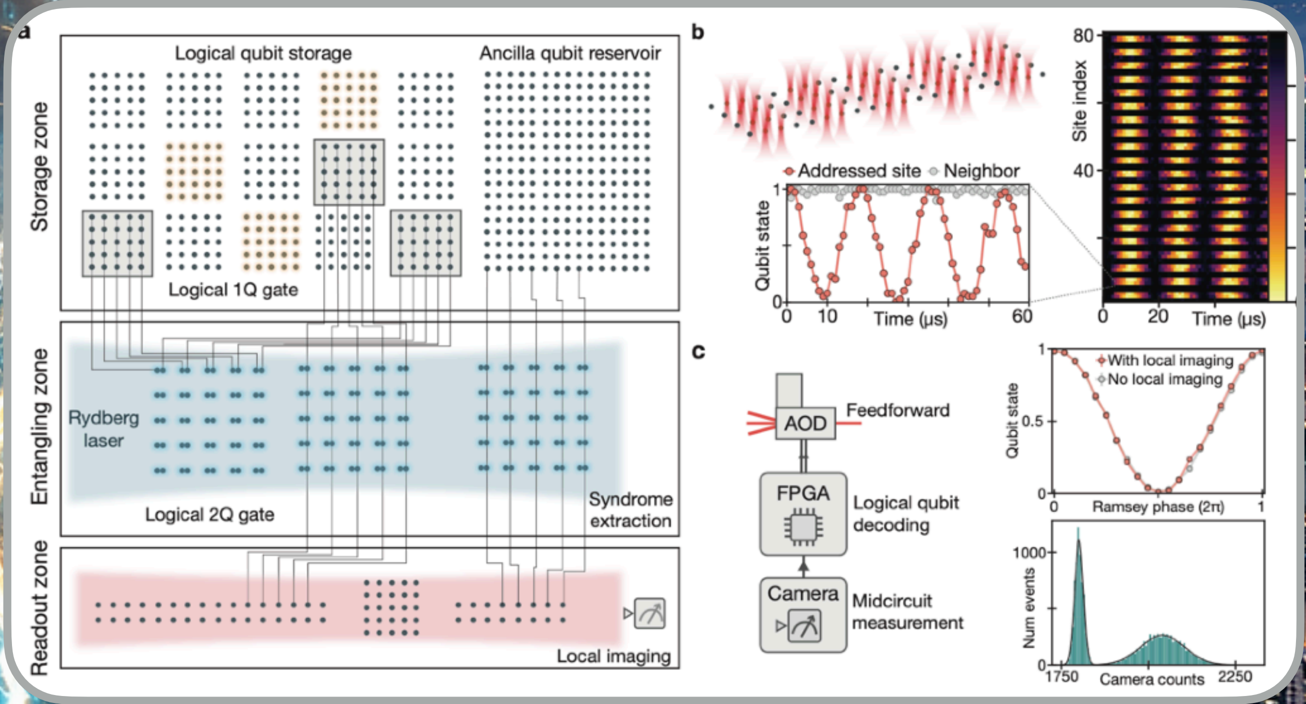
Optimising a Hamiltonian over two copies of **graph states** requires solving a 3-SAT problem with $O(\log^2 d)$ variables (with suitable penalty terms)

Leone, Oliviero, arXiv:2602.22330 (2026)

Hein, Eisert, Briegel, Phys Rev A 69, 062311 (2004)



- No efficient **magic monotones** (computing takes $\exp(\log^2 d)$ time)
- Optimizing **witnesses** is hard
- So is deciding whether **Pauli propagation** is efficient



- Think of **new magic state distillation** in QEC, but how?

- Looked at three readings of quantum resources

- Measurement is a quantum resource

- The fine line is subtle

- Looked at **three readings** of quantum resources

- **Computationally bounded observers** introduce a new perspective to physics and resources, where the observer is not a mere verifier of the quantum theory, but an integral part of the theory itself

- Measurement is a quantum resource

- The fine line is subtle

THANKS FOR THE ATTENTION

QUANTUM RESOURCES

CLASSICAL