國立臺灣大學 National Taiwan University

FOXCONN 鴻海科技集團

Joint State-Channel Decoupling

(arXiv:2409.15149)

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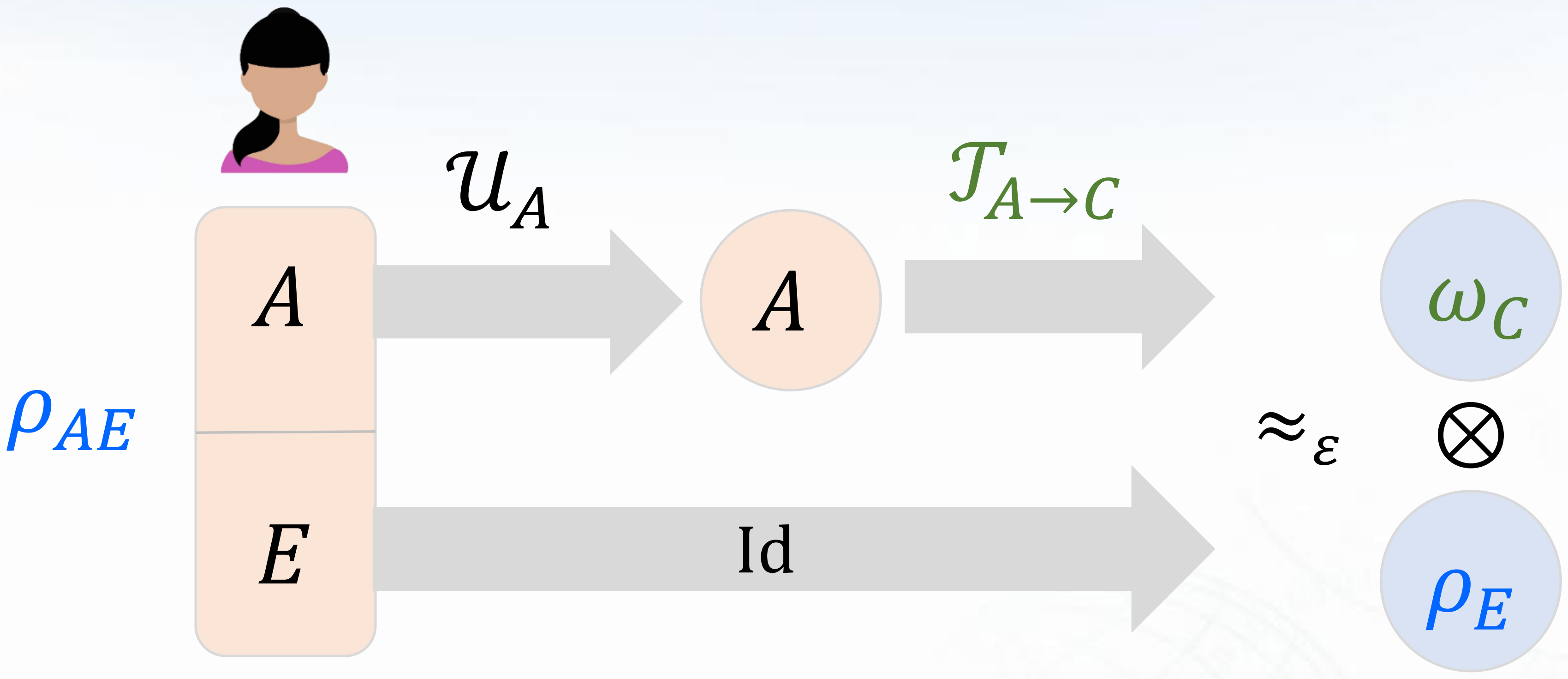
Quantum Resources, March 16–20, 2026

Quantum Information Decoupling



$$\mathbb{E}_{U_A} \frac{1}{2} \left\| \mathcal{T}_{A\to C}\left(U_A \rho_{AE} U_A^\dagger\right) - \omega_C \otimes \rho_E \right\|_1 = \text{?}$$

Notation

- Quantum systems A (with finite dimension d)

  $A' \simeq A$

  C (arbitrary)

  E (arbitrary)
- $|\Phi_{A'A}\rangle := \frac{1}{\sqrt{d}} \sum_{i,j} |i\rangle_{A'} |j\rangle_A$, the maximally entangled state between A & A'
- $\omega_{A'C} := \mathrm{id}_{A'} \otimes \mathcal{T}_{A\to C}(|\Phi_{A'A}\rangle\langle\Phi_{A'A}|)$ is the Choi state of channel $\mathcal{T}_{A\to C}$
- $\rho^U_{AE} := U_A \rho_{AE} U_A^\dagger$

$$\Delta^U := \frac{1}{2} \left\| \mathcal{T}_{A\to C}\left(\rho^U_{AE}\right) - \omega_C \otimes \rho_E \right\|_1 = \text{?}$$

Notation

Schatten α-norm:
$\|X\|_\alpha := (\mathrm{Tr}[|X|^\alpha])^{1/\alpha}$

[MDS+'13, WWY'14]

Sandwiched Rényi divergence $D_\alpha^*(\rho\|\sigma) := \frac{1}{\alpha-1}\log \mathrm{Tr}\left[\left(\sigma^{\frac{1-\alpha}{2\alpha}}\rho\sigma^{\frac{1-\alpha}{2\alpha}}\right)^\alpha\right]$

$$= \frac{\alpha}{\alpha-1}\log\left\|\sigma^{\frac{1-\alpha}{2\alpha}}\rho\sigma^{\frac{1-\alpha}{2\alpha}}\right\|_\alpha$$

Conditional sandwiched Rényi entropy

$$H_\alpha^*(A|E)_\rho := -\inf_{\sigma_E} D_\alpha^*(\rho_{AE}\|I_A\otimes\sigma_E)$$

Continuous convergence: $H_\alpha^*(A|E)_\rho \to H(A|E)_\rho$ as $\alpha\to 1$

Prior Results

Commun. Math. Phys. 269, 107–136 (2007)
Digital Object Identifier (DOI) 10.1007/s00220-006-0118-x
Communications in Mathematical Physics

Quantum State Merging and Negative Information

Michał Horodecki[1], Jonathan Oppenheim[2], Andreas Winter[3]

Open Systems & Information Dynamics | Vol. 15, No. 01, pp. 7-19 (2008)

A Decoupling Approach to the Quantum Capacity

Patrick Hayden, Michał Horodecki, Andreas Winter, and Jon Yard

arXiv > quant-ph > arXiv:0912.4495

Quantum Physics

[Submitted on 22 Dec 2009]

Single-shot Quantum State Merging

Mario Berta

PROCEEDINGS OF THE ROYAL SOCIETY A

Proc. R. Soc. A (2009) **465**, 2537–2563
doi:10.1098/rspa.2009.0202
Published online 5 June 2009

The mother of all protocols: restructuring quantum information's family tree

By Anura Abeyesinghe[1], Igor Devetak[2], Patrick Hayden[3] and Andreas Winter[4,*]

Prior Results

Commun. Math. Phys. 328, 251–284 (2014)
Digital Object Identifier (DOI) 10.1007/s00220-014-1990-4

Communications in **Mathematical Physics**

One-Shot Decoupling

Frédéric Dupuis[1], Mario Berta[1], Jürg Wullschleger[2,3], Renato Renner[1]

[1] Institute for Theoretical Physics, ETH Zurich, Zurich, Switzerland. E-mail: berta@phys.ethz.ch
[2] Department of Computer Science and Operations Research, Université de Montréal, Montreal, QC, Canada
[3] McGill University, Montreal, QC, Canada

$$\mathbb{E}_U \, \Delta^U \leq \frac{1}{2} \exp\left\{-\frac{1}{2} H_{\mathbf{2}}^{*}(A'|C)_{\omega} - \frac{1}{2} H_{\mathbf{2}}^{*}(A|E)_{\rho}\right\}$$

Implications (1/2)

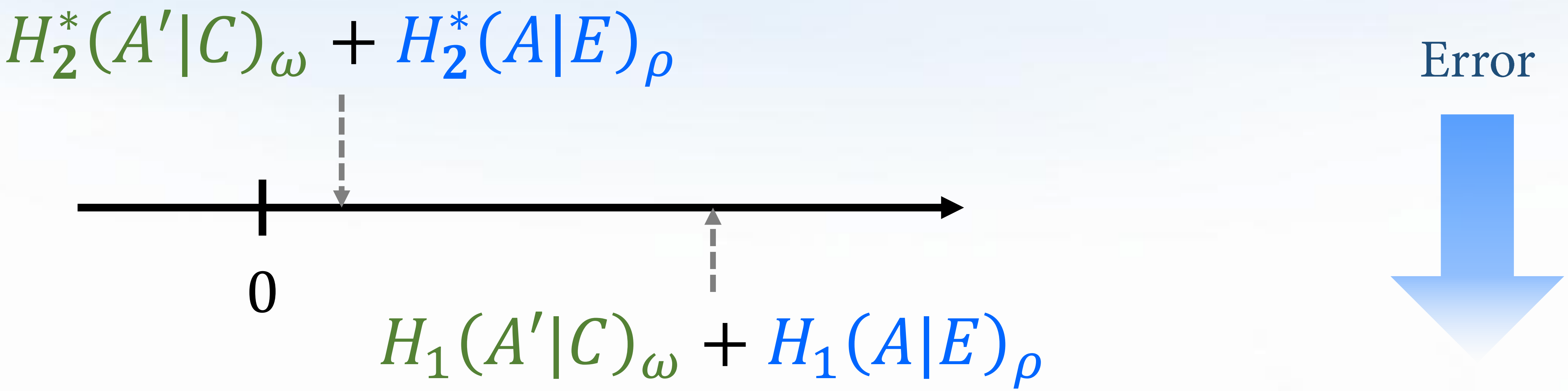
$$\mathbb{E}_U \, \Delta^U \le \frac{1}{2} \exp\left\{ -\frac{1}{2} H_2^*(A'|C)_\omega - \frac{1}{2} H_2^*(A|E)_\rho \right\}$$

- I.I.D. case: $\rho_{AE} \leftarrow \rho_{AE}^{\otimes n}$ & $\mathcal{T}_{A\to C} \leftarrow \mathcal{T}_{A\to C}^{\otimes n}$, i.e., $\omega_{A'C} \leftarrow \omega_{A'C}^{\otimes n}$

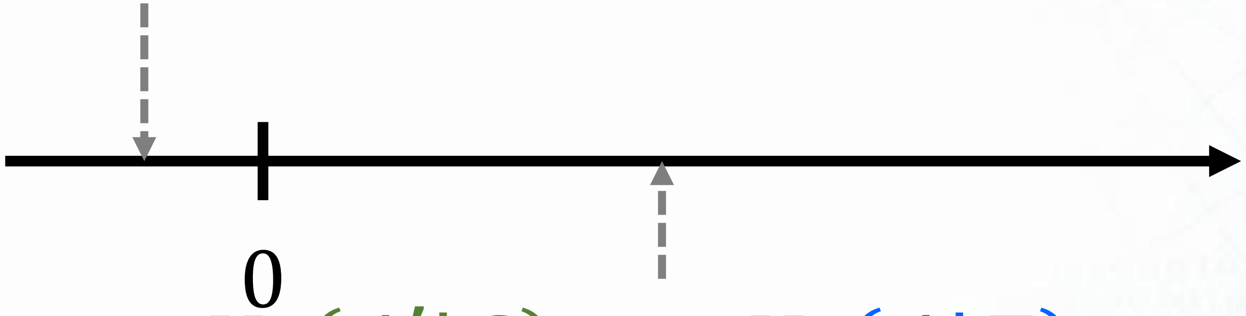
  $\Rightarrow H_2^*(A'|C)_{\omega^{\otimes n}} = nH_2^*(A'|C)_\omega$ & $H_2^*(A|E)_{\rho^{\otimes n}} = nH_2^*(A|C)_\rho$

- $\Rightarrow \mathbb{E}_{U^n} \, \Delta^{U^n}$ decays exponentially in *any* copy $n \in \mathbb{N}$

if $H_2^*(A'|C)_\omega + H_2^*(A|E)_\rho > 0$


$H_2^*(A'|C)_\omega + H_2^*(A|E)_\rho$
0
$H_1(A'|C)_\omega + H_1(A|E)_\rho$
Error

$$H_{\mathbf{2}}^{*}(A'|C)_{\omega} + H_{\mathbf{2}}^{*}(A|E)_{\rho}$$



$$H_{1}(A'|C)_{\omega} + H_{1}(A|E)_{\rho}$$

Implications (2/2)

$$\mathbb{E}_U \, \Delta^U \le \frac{1}{2} \exp\left\{-\frac{1}{2} H_{\mathbf{2}}^*(A'|C)_\omega - \frac{1}{2} H_{\mathbf{2}}^*(A|E)_\rho\right\}$$
$$\le \frac{1}{2} \exp\left\{-\frac{1}{2} H_{\min}^\varepsilon(A'|C)_\omega - \frac{1}{2} H_{\min}^\varepsilon(A|E)_\rho\right\} + 6\varepsilon$$

- AEP: $H_{\min}^\varepsilon(A|E)_{\rho^{\otimes n}} = nH(A|E)_\rho + o(n)$
- $\Rightarrow \mathbb{E}_{U^n} \, \Delta^{U^n}$ vanishes *asymptotically* as $n \to \infty$

if $H(A'|C)_\omega + H(A|E)_\rho > 0$

Prior Results: Standard Decoupling

IEEE TRANSACTIONS ON INFORMATION THEORY, VOL. 69, NO. 12, DECEMBER 2023

Privacy Amplification and Decoupling Without Smoothing

Frédéric Dupuis

- $\mathcal{T}_{A\to C} = \mathrm{Tr}_{A\setminus C}$ is partial trace with a remainder dimension $|C|$

We want $|C| \uparrow$, or equivalently, $\frac{|A|}{|C|} \downarrow$

Prior Results: Standard Decoupling

IEEE TRANSACTIONS ON INFORMATION THEORY, VOL. 69, NO. 12, DECEMBER 2023

Privacy Amplification and Decoupling Without Smoothing

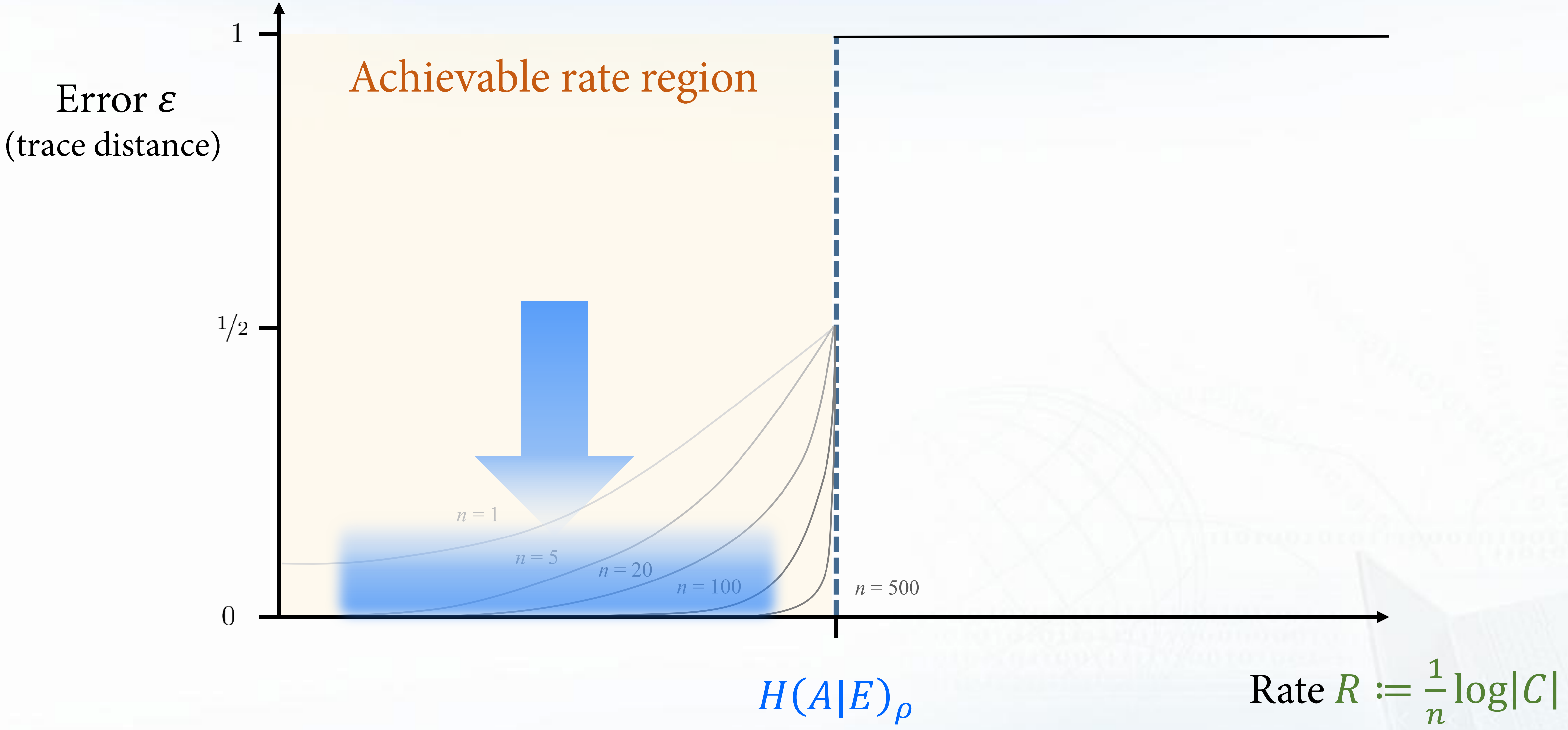
Frédéric Dupuis

- $\mathcal{T}_{A\to C} = \mathrm{Tr}_{A\setminus C}$ is partial trace with a remainder dimension $|C|$

$$\mathbb{E}_U \Delta^U \le 2^{\frac{2-\alpha}{\alpha}} \exp\left\{-\frac{\alpha-1}{\alpha}\left(H^*_\alpha(A|E)_\rho - \log|C|\right)\right\}, \forall \alpha \in [1,2]$$

I.I.D.

$$\varepsilon \leq \mathrm{e}^{-n\cdot \sup_{\alpha\in[1,2]} \frac{\alpha-1}{\alpha}\left(H^*_\alpha(A|E)_\rho - R\right)}$$



Prior Results

IEEE TRANSACTIONS ON INFORMATION THEORY, VOL. 69, NO. 12, DECEMBER 2023

Privacy Amplification and Decoupling Without Smoothing

Frédéric Dupuis

- For general decoupling channel $\mathcal{T}_{A\to C}$

$$\mathbb{E}_U \, \Delta^U \le 2^{\frac{2-\alpha}{\alpha}} \exp\left\{-\frac{\alpha-1}{\alpha}\left(H_{\mathbf{2}}^*(A'|C)_\omega + H_{\boldsymbol{\alpha}}^*(A|E)_\rho\right)\right\}, \forall \alpha \in [1,2]$$

Prior Results

Commun. Math. Phys. (2024) 405:281
Digital Object Identifier (DOI) https://doi.org/10.1007/s00220-024-05156-7

Communications in
Mathematical Physics

Check for updates

Decoupling by Local Random Unitaries without Simultaneous Smoothing, and Applications to Multi-user Quantum Information Tasks

Pau Colomer[1,2,4], **Andreas Winter**[1,3,4]

$$\mathbb{E}_U \, \Delta^U \leq 2^{\frac{2-\alpha}{\alpha}} \exp\left\{ -\frac{1}{2} H_2^{\varepsilon}(A'|C)_\omega - \frac{\alpha-1}{\alpha} H_{\alpha}^{*}(A|E)_\rho \right\} + \varepsilon'$$

Main Question

- Dupuis (2023) "Rényifies" the **state** part:

$$H_2^*(A|E)_\rho \dashrightarrow H_\alpha^*(A|E)_\rho \ \forall\alpha \in [1,2]$$

but the channel part $H_2^*(A'|C)_\omega$ remains.

- The bound implies that the decoupling error decays exponentially only if

$$H_2^*(A'|C)_\omega + H_1^*(A|E)_\rho > 0$$

How to simultaneously "Rényify" both the **state** and **channel** parts?

Main Results

A One-Shot Achievability Bound

- For any states $\omega_{A'C}$ & ρ_{AE} and any $\alpha \in [1,2]$

Theorem. $\mathbb{E}_U \, \Delta^U \leq 3^{\frac{1-\alpha}{\alpha}} \exp\left\{-\frac{\alpha-1}{\alpha}\left(H_\alpha^*(A'|C)_\omega + H_\alpha^*(A|E)_\rho\right)\right\}$

- Decays exponentially for *any* number of copies $n \in \mathbb{N}$
- An achievable error exponent

$$\sup_{\alpha\in[1,2]} \frac{\alpha-1}{\alpha}\left(H_\alpha^*(A'|C)_\omega + H_\alpha^*(A|E)_\rho\right) > 0 \Leftrightarrow H(A'|C)_\omega + H(A|E)_\rho > 0$$

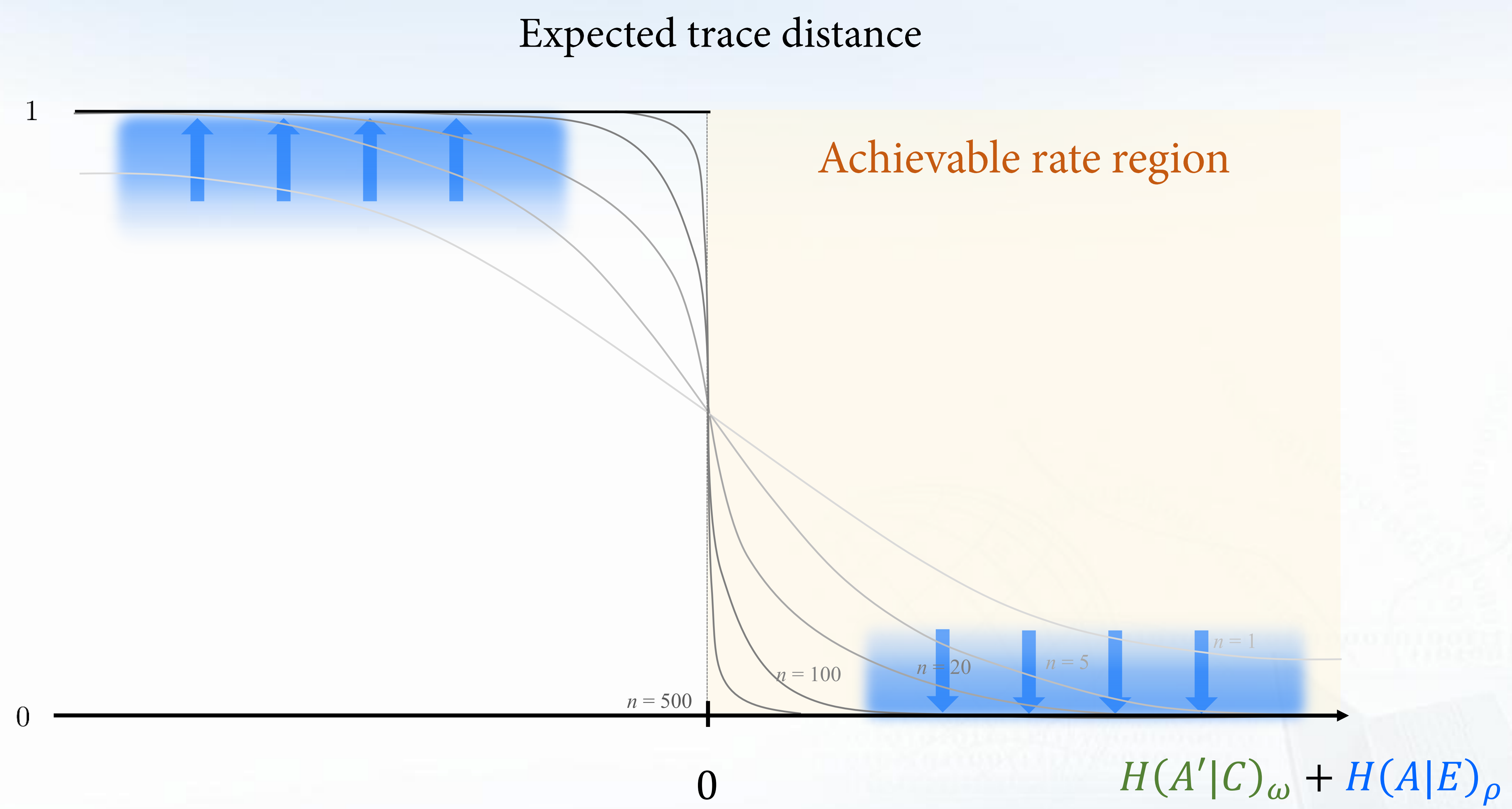
A One-Shot Strong Converse Bound

- For any states $\omega_{A'C}$ & ρ_{AE} and any $\alpha \in (0,1)$

$$\mathbb{E}_U \Delta^U \geq 1 - 3\exp\left\{(1-\alpha)\left(H_\alpha^\downarrow(A'|C)_\omega + H_\alpha^\downarrow(A|E)_\rho\right)\right\}$$

- $H_\alpha^\downarrow(A|E)_\rho \ := \frac{1}{1-\alpha} \log \mathrm{Tr}[\rho_{AE}^\alpha (I_A \otimes \rho_E)^{1-\alpha}]$
- A strong converse exponent

$$\sup_{\alpha\in(0,1)} (\alpha-1)\left(H_\alpha^\downarrow(A'|C)_\omega + H_\alpha^\downarrow(A|E)_\rho\right) > 0 \Leftrightarrow H(A'|C)_\omega + H(A|E)_\rho < 0$$



Expected trace distance
1
0
Achievable rate region
n = 1
n = 5
n = 20
n = 100
n = 500
0
$H(A'|C)_\omega + H(A|E)_\rho$

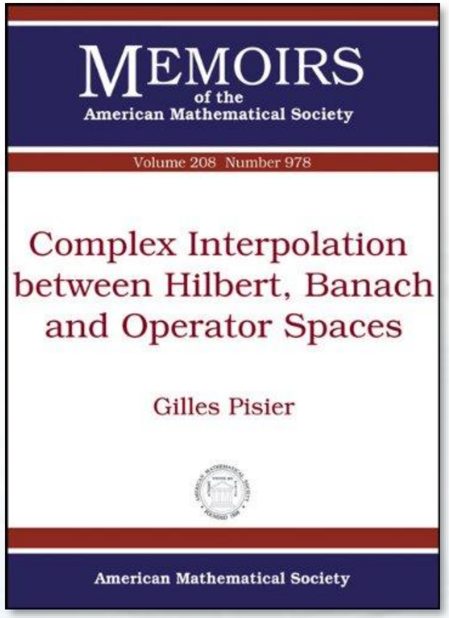
Proof Idea (Achievability)

Main Technique: Norm Interpolation Inequality

- Schatten p-norm:

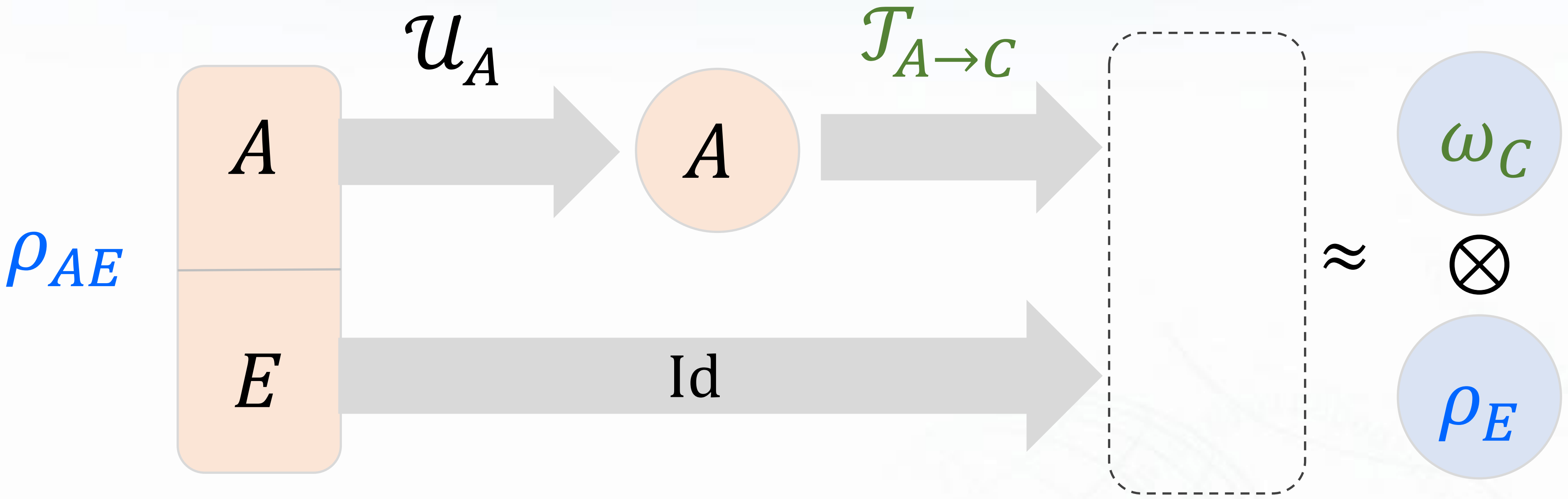
$$\|A\|_p \leq \|A\|_{p_0}^{1-\theta} \|A\|_{p_1}^{\theta}, \ \frac{1}{p} = \frac{1-\theta}{p_0} + \frac{\theta}{p_1}, \theta \in [0,1]$$

- Log-convexity of the norm: $\frac{1}{p} \mapsto \|A\|_p$
- Riesz-Thorin Interpolation Theorem
- Hadamard's Three-Line Theorem
- Hölder's inequality



Crucial Observation

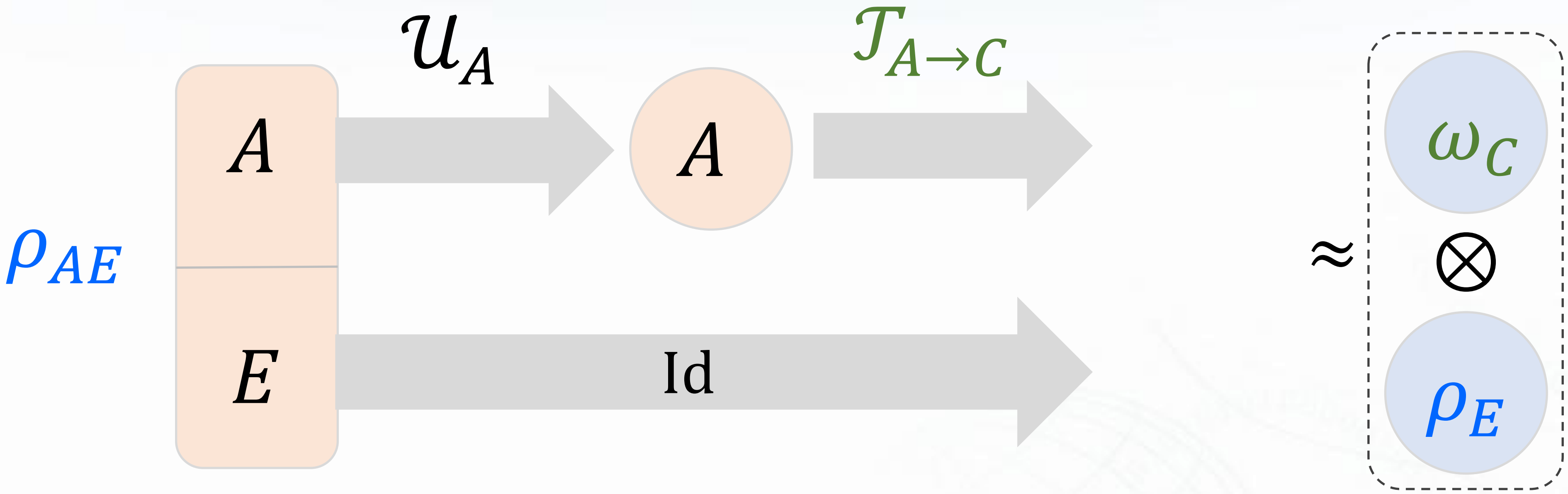
$$\langle \Phi_{A'A} | X \otimes Y | \Phi_{A'A} \rangle = \frac{1}{|A|} \mathrm{Tr}_{A'A}[X^T Y]$$



$$\mathcal{T}_{A\to C}\left(U_A \rho_{AE} U_A^\dagger\right) = d \left\langle \Phi_{A'A} \middle| \omega_{A'C} \otimes U_A \rho_{AE} U_A^\dagger \middle| \Phi_{A'A} \right\rangle$$

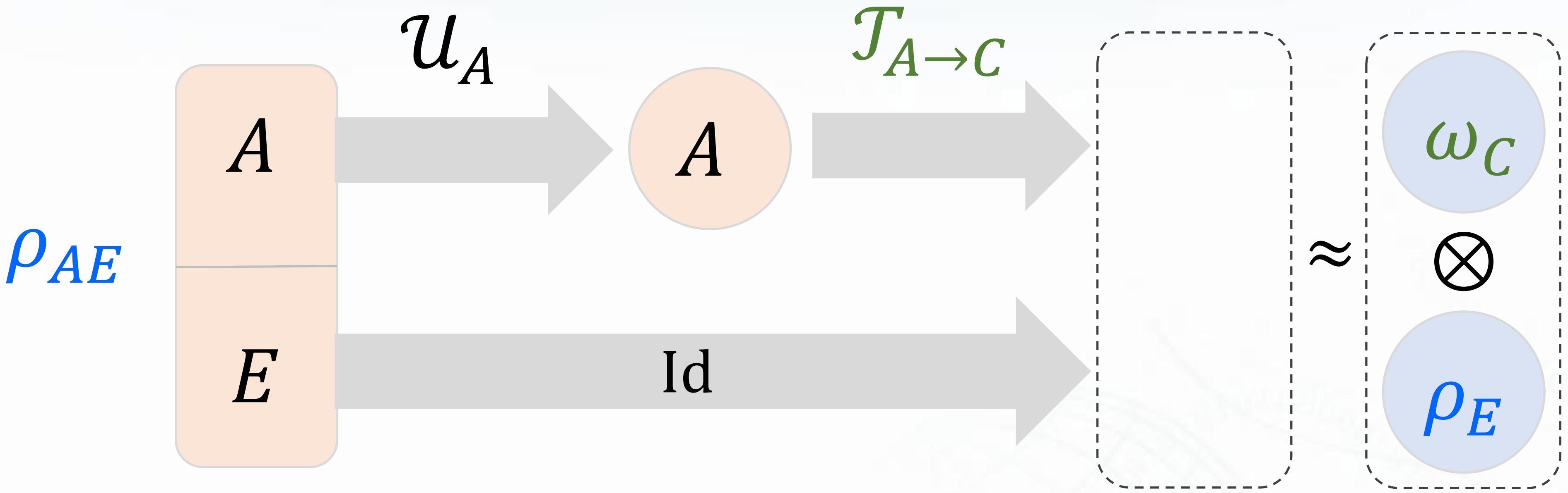
Crucial Observation

$$\langle \Phi_{A'A} | X \otimes Y | \Phi_{A'A} \rangle = \frac{1}{|A|} \mathrm{Tr}_{A'A}[X^T Y]$$



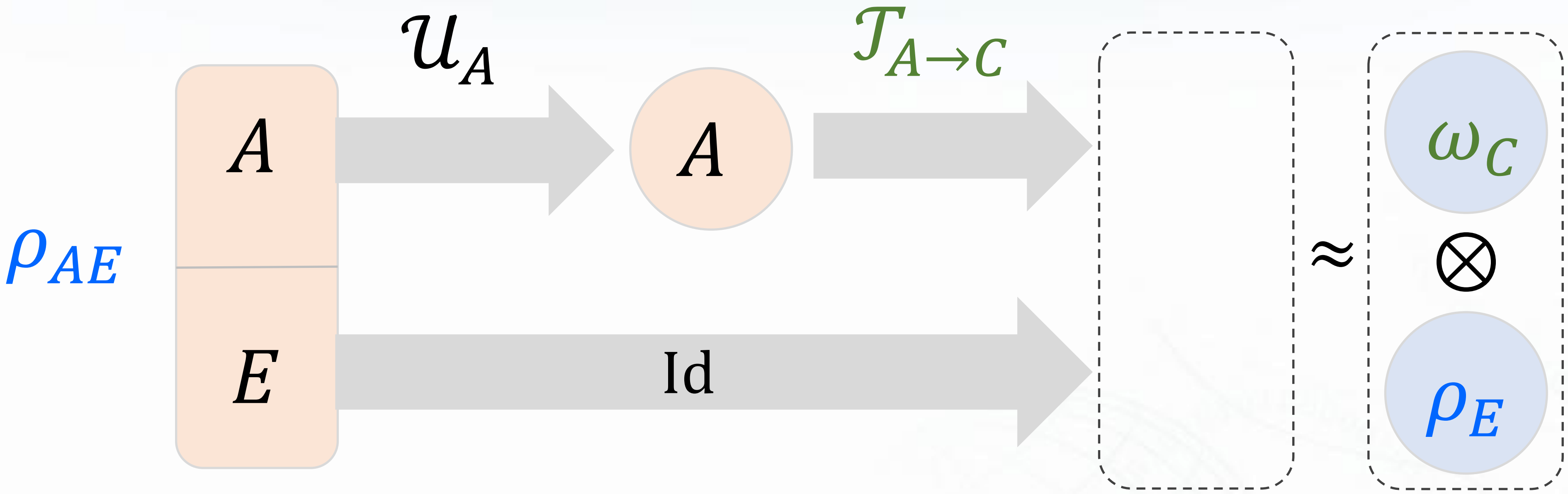
$$\omega_C \otimes \rho_E = \mathbb{E}_U \, d \left\langle \Phi_{A'A} \middle| \omega_{A'C} \otimes U_A \rho_{AE} U_A^\dagger \middle| \Phi_{A'A} \right\rangle$$

Analysis from The Functional Perspective



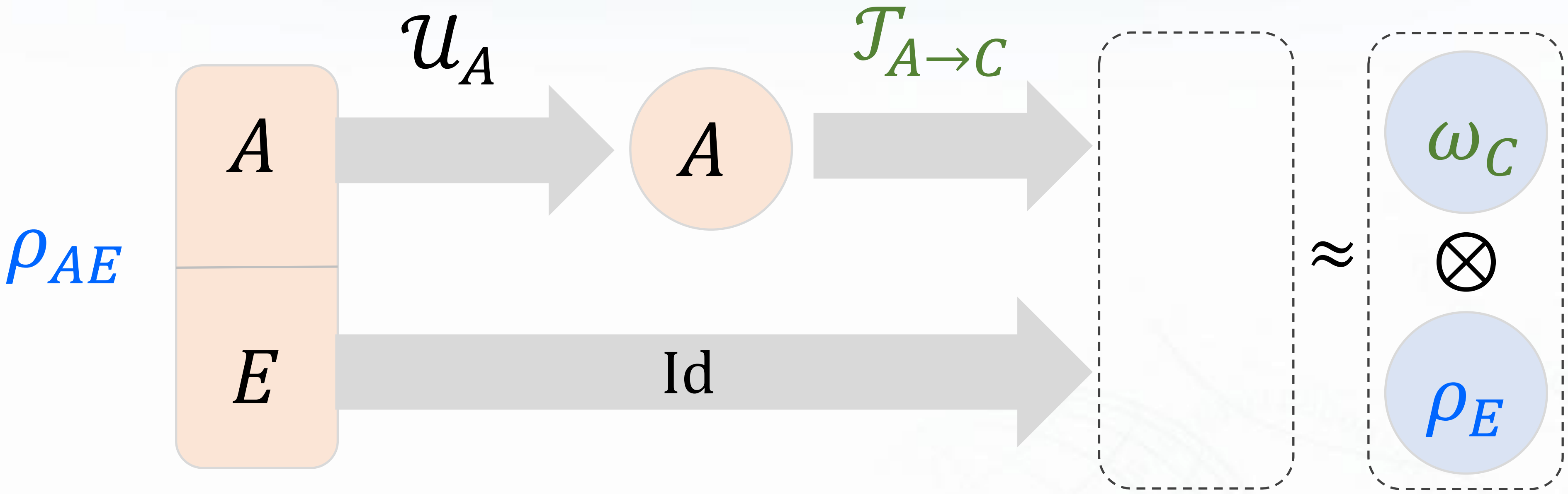
$$(\mathcal{T}_{A\to C}, \rho_{AE}) \mapsto d \left\langle \Phi_{A'A} \middle| \omega_{A'C} \otimes U_A \rho_{AE} U_A^\dagger \middle| \Phi_{A'A} \right\rangle$$

Analysis from The Functional Perspective



$$\left(\omega_{A'C}, \rho_{AE}\right) \mapsto d \left\langle \Phi_{A'A} \middle| \omega_{A'C} \otimes U_A \rho_{AE} U_A^\dagger \middle| \Phi_{A'A} \right\rangle$$

Analysis from The Functional Perspective



$$\boldsymbol{Y}_{AA'CE} \mapsto d\left\langle \Phi_{A'A} \middle| U_A \boldsymbol{Y}_{AA'CE} U_A^\dagger \middle| \Phi_{A'A} \right\rangle$$

Decoupling Map

Operator-valued
Random variable

$$\Theta: \mathcal{B}(AA'CE) \to L_\infty(\mathbb{U}(A), \mathcal{B}(CE))$$

$$\Theta(\boldsymbol{Y}_{AA'CE}) := d\left\langle \Phi_{A'A} \middle| U_A \boldsymbol{Y}_{AA'CE} U_A^\dagger \middle| \Phi_{A'A} \right\rangle$$

$$\mathbb{E}_U \Theta(Y_{AA'CE}) = \mathbb{E}_U\, d\left\langle \Phi_{A'A} \middle| U_A Y_{AA'CE} U_A^\dagger \middle| \Phi_{A'A} \right\rangle$$

$$= d\left\langle \Phi_{A'A} \middle| \frac{I_A}{d} \otimes Y_{A'CE} \middle| \Phi_{A'A} \right\rangle$$

Mean-Deviation Bound

- For any bounded operator $\boldsymbol{Y}_{\boldsymbol{AA'CE}} \in \mathcal{B}(AA'CE)$,

$$\mathbb{E}_U\, \Delta^U(\boldsymbol{Y}_{\boldsymbol{AA'CE}}) := \frac{1}{2}\mathbb{E}_U \left\| \Theta(\boldsymbol{Y}_{\boldsymbol{AA'CE}}) - \mathbb{E}_U \Theta(\boldsymbol{Y}_{\boldsymbol{AA'CE}}) \right\|_1 = ❓$$

- Decoupling

$$\mathbb{E}_U \Delta^U(\omega_{A'C} \otimes \rho_{AE}) := \frac{1}{2}\mathbb{E}_U \left\| \Theta(\omega_{A'C} \otimes \rho_{AE}) - \mathbb{E}_U \Theta(\omega_{A'C} \otimes \rho_{AE}) \right\|_1 = ❓$$

Proof Sketch

Hölder's inequality, $\frac{1}{\alpha} + \frac{1}{\alpha'} = 1$

$$\mathbb{E}_U \left\|\Theta(\boldsymbol{Y}) - \mathbb{E}_U\Theta(\boldsymbol{Y})\right\|_1$$

$$\leq \mathbb{E}_U \left\|\sigma_{CE}^{\frac{1}{2\alpha'}}\right\|_{2\alpha'} \left\|\sigma_{CE}^{-\frac{1}{2\alpha'}}[\Theta(\boldsymbol{Y}) - \mathbb{E}_U\Theta(\boldsymbol{Y})]\sigma_{CE}^{-\frac{1}{2\alpha'}}\right\|_{\alpha} \left\|\sigma_{CE}^{\frac{1}{2\alpha'}}\right\|_{2\alpha'}$$

$$= \mathbb{E}_U \left\|\sigma_{CE}^{-\frac{1}{2\alpha'}}[\Theta(\boldsymbol{Y}) - \mathbb{E}_U\Theta(\boldsymbol{Y})]\sigma_{CE}^{-\frac{1}{2\alpha'}}\right\|_{\alpha}$$

$$= \mathbb{E}_U \left\|\Theta\left(\sigma_{CE}^{-\frac{1}{2\alpha'}}\boldsymbol{Y}\sigma_{CE}^{-\frac{1}{2\alpha'}}\right) - \mathbb{E}_U\Theta\left(\sigma_{CE}^{-\frac{1}{2\alpha'}}\boldsymbol{Y}\sigma_{CE}^{-\frac{1}{2\alpha'}}\right)\right\|_{\alpha}$$

$$=: \mathbb{E}_U \left\|\Theta(\widetilde{\boldsymbol{Y}}) - \mathbb{E}_U\Theta(\widetilde{\boldsymbol{Y}})\right\|_{\alpha} = \leq \mathbb{E}_U\left(\mathbb{E}_U\left\|\Theta(\widetilde{\boldsymbol{Y}}) - \mathbb{E}_U\Theta(\widetilde{\boldsymbol{Y}})\right\|_{\alpha}^{\alpha}\right)^{1/\alpha}$$

Proof Sketch

$$\left(\mathbb{E}_U \left\| \Theta(\widetilde{\boldsymbol{Y}}) - \mathbb{E}_U \Theta(\widetilde{\boldsymbol{Y}}) \right\|_\alpha^\alpha \right)^{1/\alpha}$$

Proof Sketch

$$\left(\mathbb{E}_U \left\| \Theta(\widetilde{Y}) - \mathbb{E}_U \Theta(\widetilde{Y}) \right\|_\alpha^\alpha\right)^{1/\alpha}$$

Proof Sketch

$$\left(\mathbb{E}_U \left\|\Theta(\widetilde{Y}) - \mathbb{E}_U \Theta(\widetilde{Y})\right\|_\alpha^\alpha\right)^{1/\alpha} \overset{?}{\leq} c_\alpha \|\widetilde{Y}\|_\alpha = c_\alpha \left\| \sigma_{CE}^{-\frac{1}{2\alpha'}} Y \sigma_{CE}^{-\frac{1}{2\alpha'}} \right\|_\alpha$$

$$\left\|\Theta(\widetilde{Y}) - \mathbb{E}_U \Theta(\widetilde{Y})\right\|_{L_\alpha(S_\alpha)}$$

$$\to e^{\frac{\alpha-1}{\alpha} H_\alpha^*(A'A|CE)_Y}$$

$$\|\boldsymbol{\Theta} - \mathbb{E}_U \boldsymbol{\Theta} \colon S_\alpha \to L_\alpha(S_\alpha)\| \coloneqq \sup_{\widetilde{Y} \neq 0} \frac{\|\Theta(\widetilde{Y}) - \mathbb{E}_U \Theta(\widetilde{Y})\|_{L_\alpha(S_\alpha)}}{\|\widetilde{Y}\|_\alpha} \leq ?$$

Proof Sketch

1. $(\alpha = 1)$ By triangle inequality:

$$\Rightarrow \|\boldsymbol{\Theta} - \mathbb{E}_U \boldsymbol{\Theta}\colon S_1 \to L_1(S_1)\| \leq 2$$

2. $(\alpha = 2)$ By twirling:

$$\Rightarrow \|\boldsymbol{\Theta} - \mathbb{E}_U \boldsymbol{\Theta}\colon S_2 \to L_2(S_2)\| \leq \sqrt{\frac{d^2}{d^2-1}} \leq \frac{2}{\sqrt{3}}, \forall d \geq 2$$

Proof Sketch

3. ($\alpha \in [1,2]$) **Interpolation inequality:**

$$\Rightarrow \|\mathbf{\Theta} - \mathbb{E}_U \mathbf{\Theta}: S_\alpha \to L_\alpha(S_\alpha)\|$$

$$\le \underbrace{\|\mathbf{\Theta} - \mathbb{E}_U \mathbf{\Theta}: S_1 \to L_1(S_1)\|}_{2}{}^{1-\theta} \cdot \underbrace{\|\mathbf{\Theta} - \mathbb{E}_U \mathbf{\Theta}: S_2 \to L_2(S_2)\|}_{\frac{2}{\sqrt{3}}}{}^{\theta}$$

$$= 2 \cdot 3^{\frac{1-\alpha}{\alpha}}$$

$$\frac{1}{\alpha} = \frac{1-\theta}{1} + \frac{\theta}{2}, \qquad \theta = 2\frac{\alpha - 1}{\alpha}$$

□

Main Result (Joint Interpolation)

Theorem. For any bounded operator $\boldsymbol{Y}_{\boldsymbol{AA'CE}} \in \mathcal{B}(AA'CE)$,

$$\mathbb{E}_U\, \Delta^U(\boldsymbol{Y}_{\boldsymbol{AA'CE}}) \le 3^{\frac{1-\alpha}{\alpha}} \left\| \sigma_{CE}^{-\frac{1}{2\alpha'}} \boldsymbol{Y} \sigma_{CE}^{-\frac{1}{2\alpha'}} \right\|_{\alpha}, \qquad \forall \sigma_{CE}, \alpha \in [1,2]$$

In particular, for any positive operator $\tau_{AA'CE}$,

$$\mathbb{E}_U\, \Delta^U(\boldsymbol{\tau}_{\boldsymbol{AA'CE}}) \le 3^{\frac{1-\alpha}{\alpha}} \mathrm{e}^{\frac{\alpha-1}{\alpha} H_\alpha^*(A'A|CE)_\tau}, \qquad \forall \alpha \in [1,2]$$

Back to decoupling

Lemma (Additivity [Müller-Lennert et al. (2013), Beigi (2013)]**).**

In particular, for any positive $\omega_{A'C}$ and ρ_{AE},

$$H_\alpha^*(A'A|CE)_{\omega\otimes\rho} = H_\alpha^*(A'|C)_\omega + H_\alpha^*(A|E)_\rho$$

Then, for any $\alpha \in [1,2]$,

$$\mathbb{E}_U \Delta^U\left(\omega_{A'C} \otimes \rho_{AE}\right) \leq 3^{\frac{1-\alpha}{\alpha}} \mathrm{e}^{\frac{1-\alpha}{\alpha}\left(H_\alpha^*(A'|C)_\omega + H_\alpha^*(A|E)_\rho\right)}$$

Back to decoupling

Corollary.

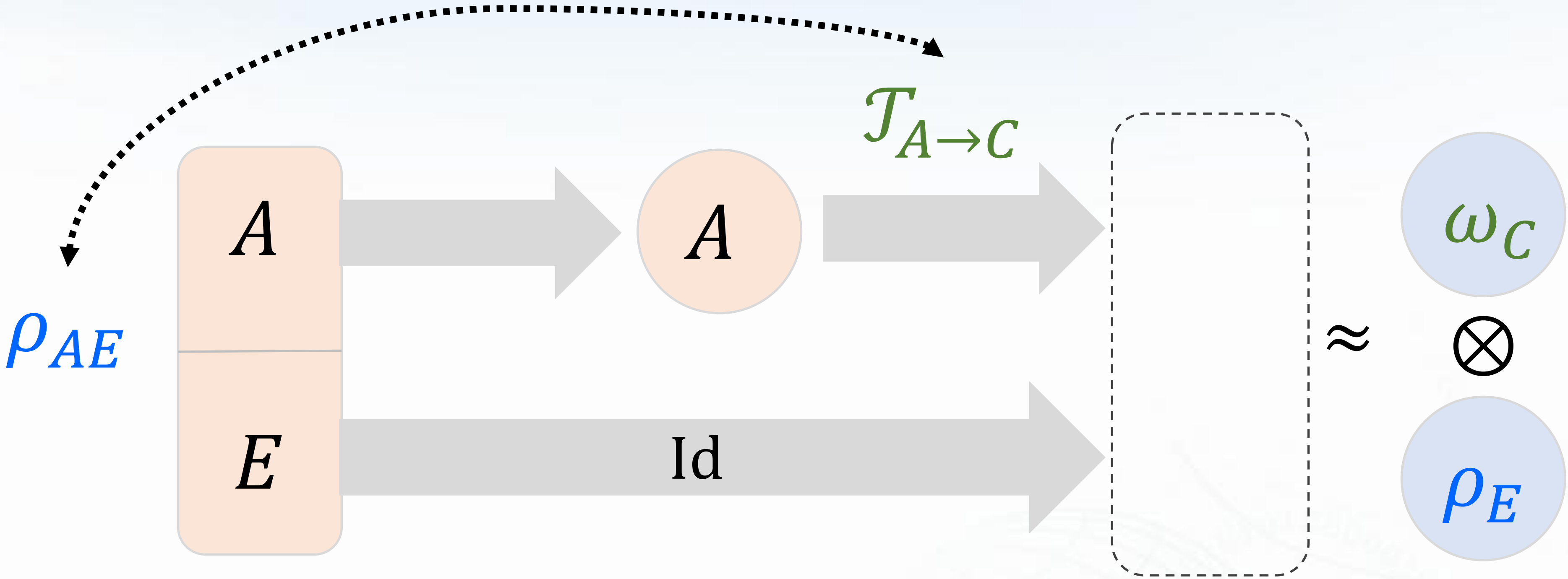
For *any* integer n,

$$\mathbb{E}_{U^n} \Delta^{U^n}\left(\omega_{A'C}^{\otimes n} \otimes \rho_{AE}^{\otimes n}\right) \leq \mathrm{e}^{-n \sup_{\alpha \in [1,2]} \frac{\alpha-1}{\alpha}\left(H_\alpha^*(A'|C)_\omega + H_\alpha^*(A|E)_\rho\right)}$$

The error exponent $> 0 \iff H(A'|C)_\omega + H(A|E)_\rho > 0$

Implications

Mathematically, **Channel** and **State** can be entangled!

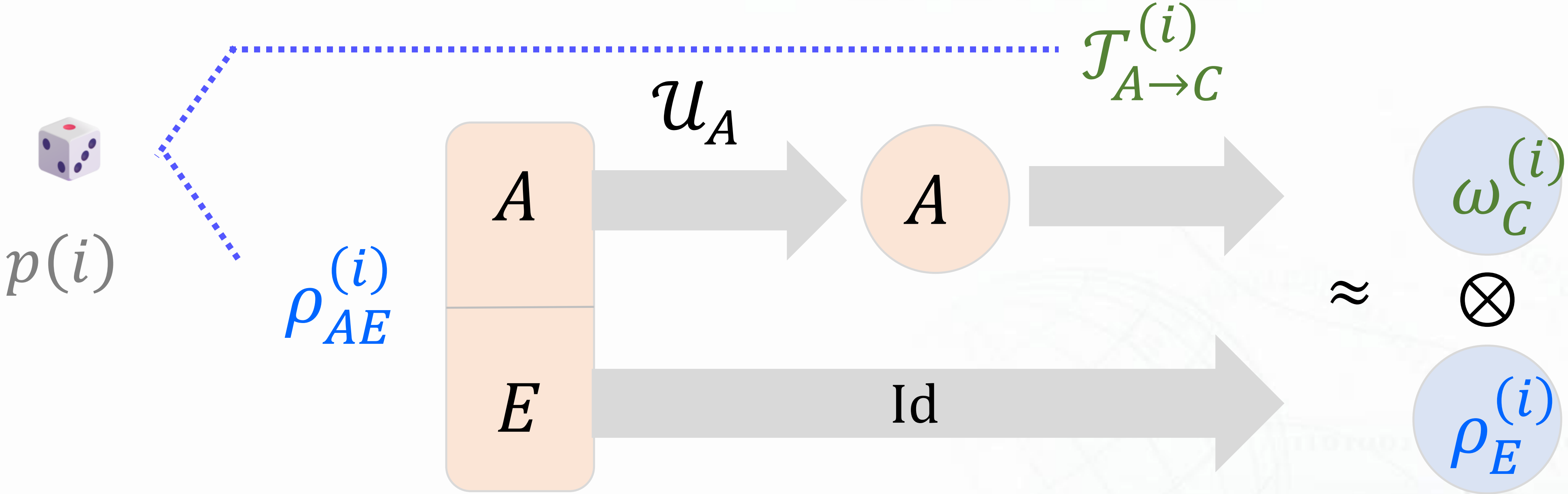


Theorem. For any positive operator $\boldsymbol{\tau}_{AA'CE}$,

$$\mathbb{E}_U \Delta^U\left(\boldsymbol{\tau}_{AA'CE}\right) \leq 3^{\frac{1-\alpha}{\alpha}} \mathrm{e}^{\frac{\alpha-1}{\alpha} H_\alpha^*(A'A|CE)_\tau}, \qquad \forall \alpha \in [1,2]$$

Randomness-Assisted Decoupling

- Let $\left\{p(i), \rho_{AE}^{(i)}, \mathcal{T}_{A\to C}^{(i)}\right\}$ be an ensemble of state-channel pairs.



$$\mathbb{E}_{U_A} \frac{1}{2} \left\| \sum_i p(i) \mathcal{T}_{A\to C}^{(i)} \left(U_A \rho_{AE}^{(i)} U_A^\dagger \right) - \omega_C^{(i)} \otimes \rho_E^{(i)} \right\|_1$$

Recent Results

Recent Results

arXiv > quant-ph > arXiv:2603.04493

Quantum Physics

[Submitted on 4 Mar 2026 (v1), last revised 12 Mar 2026 (this version, v2)]

Rethinking quantum smooth entropies: Tight one-shot analysis of quantum privacy amplification

Bartosz Regula, Marco Tomamichel

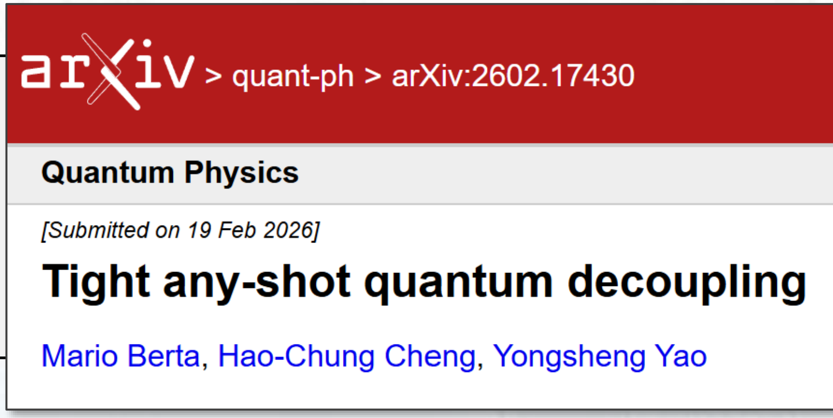
- Strengthening Dupuis (2023) to **measured** Rényi entropies

Recent Results (Talk on Thursday Morning)

Trace distance: $\mathbb{E}_U \frac{1}{2}\left\|\mathcal{T}_{A\to C}\left(\rho_{AE}^U\right) - \omega_C \otimes \rho_E\right\|_1 \leq 3^{\frac{1-\alpha}{\alpha}} \mathrm{e}^{\frac{1-\alpha}{\alpha}\left(H_\alpha^*(A'|C)_\omega + H_\alpha^*(A|E)_\rho\right)}$

- Relative entropy/purified distance criterion

Relative entropy: $\mathbb{E}_U D\left(\mathcal{T}_{A\to C}\left(\rho_{AE}^U\right)\|\omega_C \otimes \rho_E\right) \leq$?



Conclusions

- One-shot achievability bound for $H(A'|C)_\omega + H(A|E)_\rho > 0$

$$\sup_{\alpha\in[1,2]} \frac{\alpha-1}{\alpha}\left(H_\alpha^*(A'|C)_\omega + H_\alpha^*(A|E)_\rho\right)$$

- One-shot strong converse bound for $H(A'|C)_\omega + H(A|E)_\rho < 0$

$$\sup_{\alpha\in(0,1)} (\alpha-1)\left(H_\alpha^\downarrow(A'|C)_\omega + H_\alpha^\downarrow(A|E)_\rho\right)$$

- Asymptotic tightness?
- Applications of decoupling for correlated state & channel?

arXiv:2409.15149