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QUANTUM NON-MARKOVIANITY IS A FUNDAMENTAL THERMODYNAMIC RESOURCE

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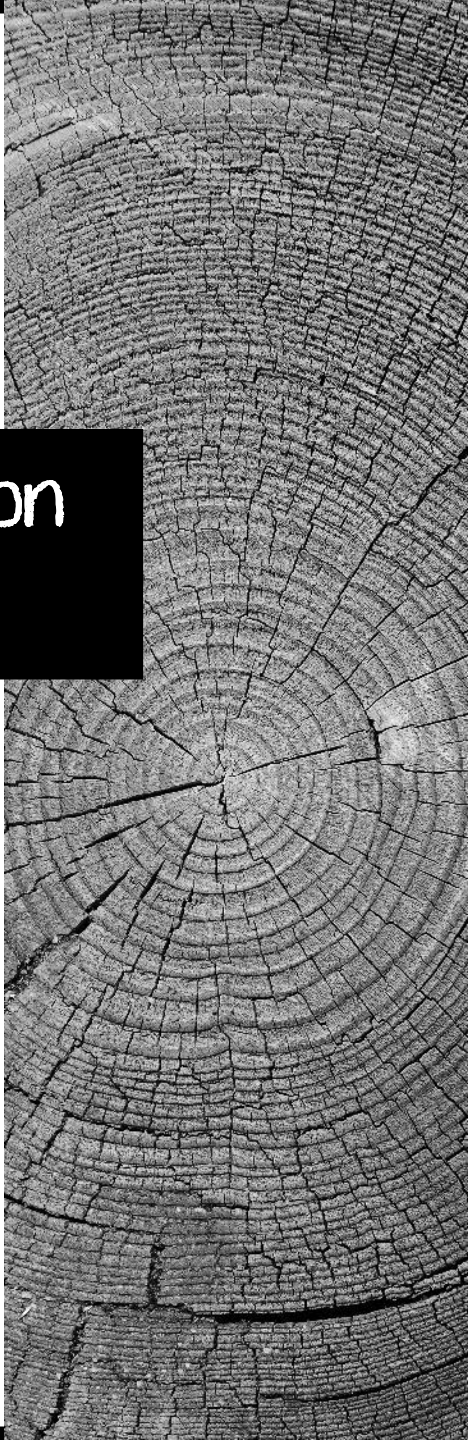
Phys Rev Letters
134, 200401 (2025)



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Look out for more

Gaussian measured max-relative entropy

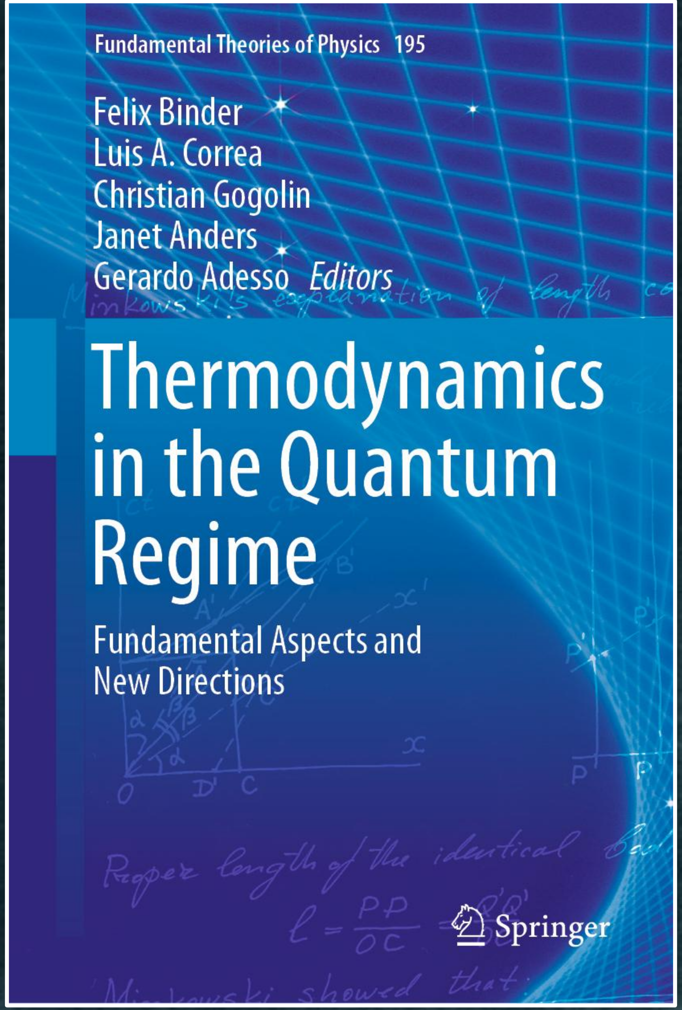
Complexity-energy trade-off in quantum phase estimation

Quantum resource theory for surface crack detection

Quantum Thermodynamics

Studies work, heat, and energy transfer in the quantum world

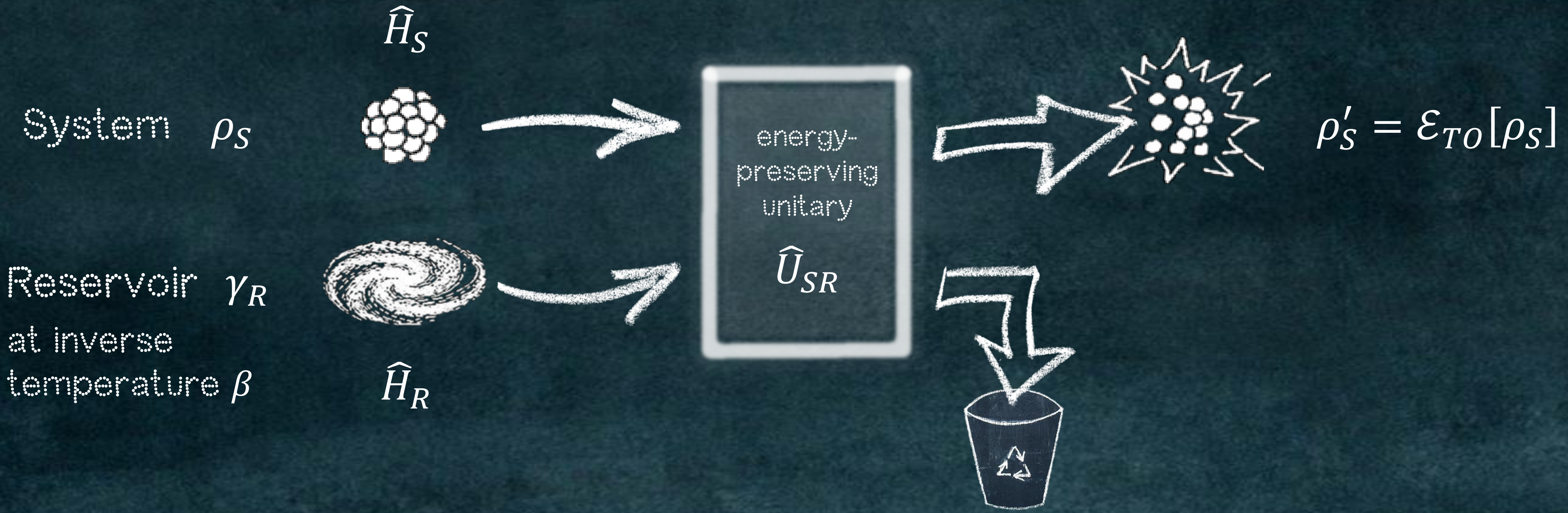
Determines fundamental limitations on nanoscale heaters and coolers



Explores the ultimate possibilities for quantum information as fuel

Can be formulated as a resource theory of quantum states out of equilibrium

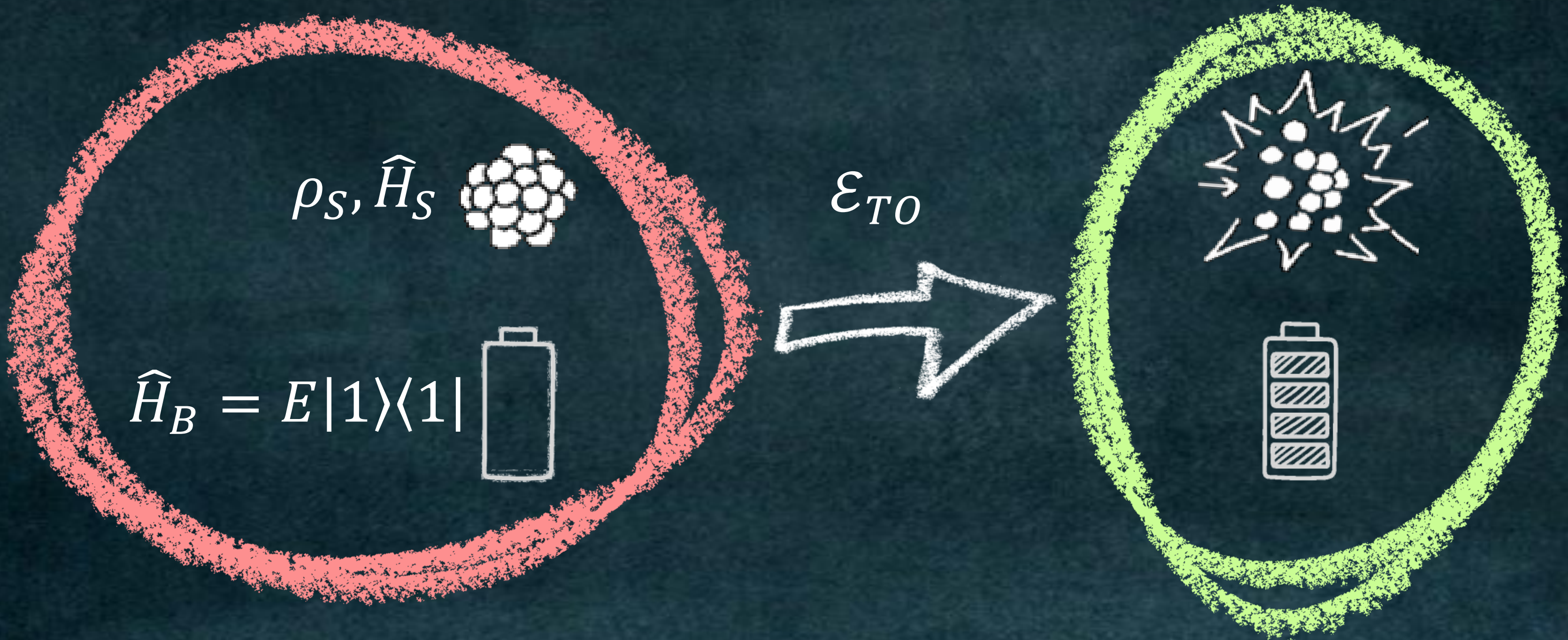
Resource Theory of Athermality



- **Thermal operations:** $\mathcal{E}_{TO}[\rho_S] = \mathrm{Tr}_R\left[\hat{U}_{SR}(\rho_S \otimes \gamma_R)\hat{U}_{SR}^{\dagger}\right]$ with $\left[\hat{U}_{SR}, \hat{H}_S + \hat{H}_R\right] = 0$
- **Thermal states:** Gibbs states $\gamma_X = \frac{\exp(-\beta \hat{H}_X)}{\mathrm{Tr}[\exp(-\beta \hat{H}_X)]}$ for system X with Hamiltonian $\hat{H}_X$

Work extraction in quantum states

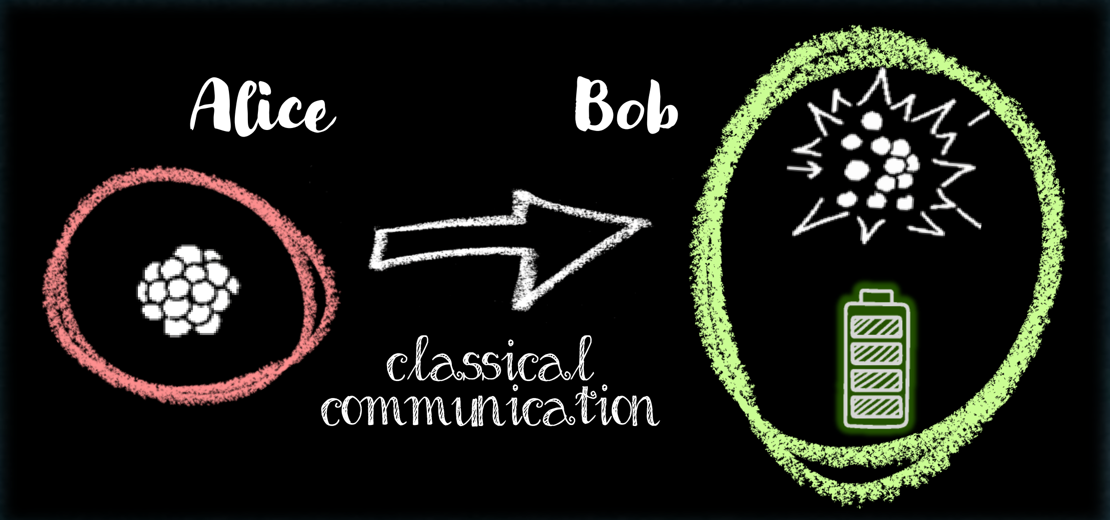
Brandao et al
PRL 111, 250404 (2013)

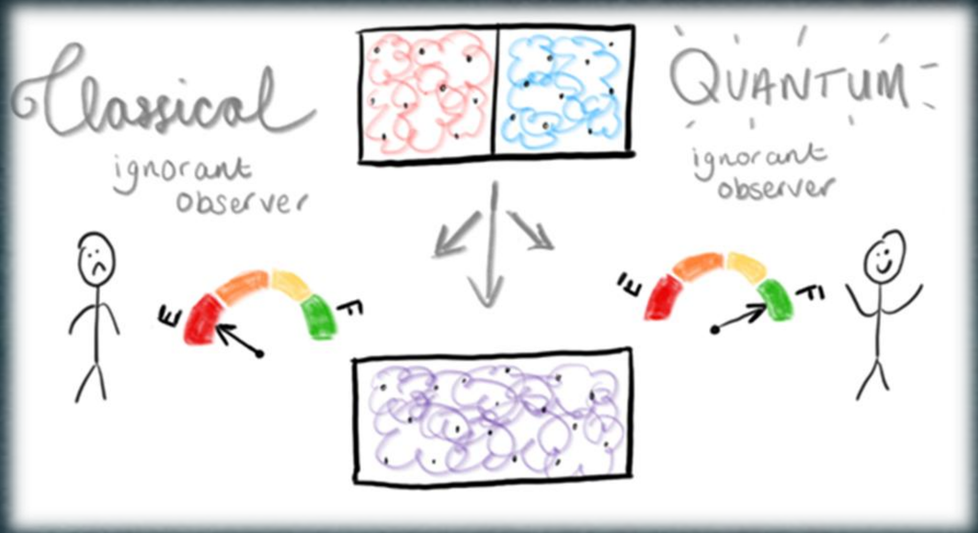


- **Extractable work:** $W(\rho_S) = \sup_{\mathcal{E}_{TO}} RE : \mathcal{E}_{TO}\left[\rho_S^{\otimes n} \otimes |0\rangle\langle 0|_B^{\otimes [Rn]}\right] \xrightarrow[n\to\infty]{} |1\rangle\langle 1|_B^{\otimes [Rn]}$
- **Athermality as resource:** $W(\rho_S) \to \Delta F(\rho_S) = \beta^{-1} S(\rho_S || \gamma_S)$ (relative entropy)
- **Reversibility:** Work cost of creating $\rho_S^{\otimes n}$ is also asymptotically given by $W(\rho_S)$

Morris, Lami, GA, Phys Rev Lett 122, 130601 (2019)

Yadin, Morris, GA, Nat Comm 12, 1471 (2021)





Quantum >> Classical thermodynamic agents

Quantum correlations and ignorance are extra resources for work extraction

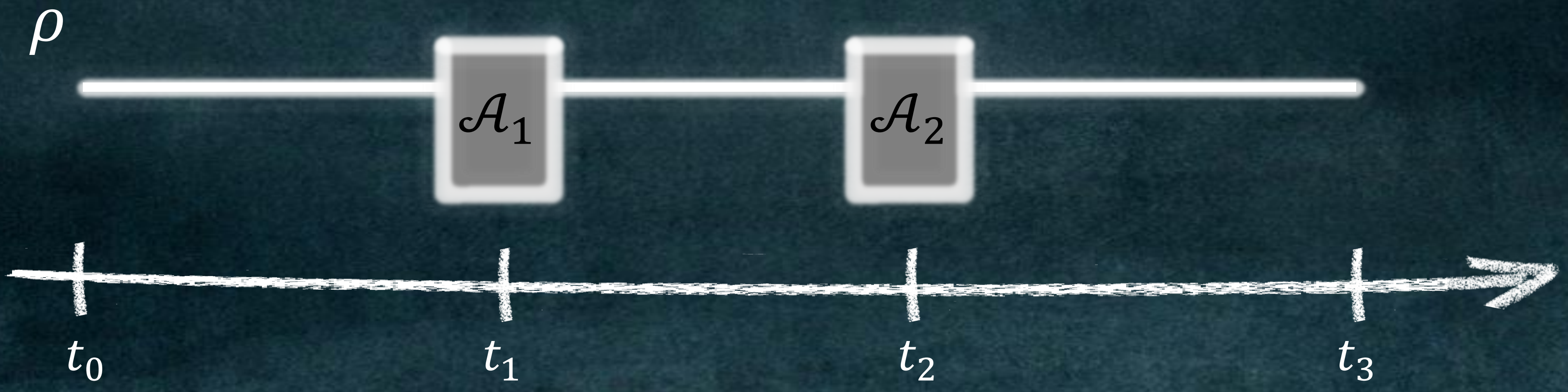
Work extraction from channels

Navascues, García-Pintos, PRL 115,010405 (2015)
Faist et al, PRL 122 200601 (2019)

- **Net extractable work:** $W(\mathcal{E}) = \max_{\rho}\left[W(\mathcal{E}(\rho)) - W(\rho)\right]$ (dynamical resource)
- **Reversibility:** Work cost of implementing the channel $\mathcal{E}$ also converges to $W(\mathcal{E})$

General multitime quantum processes

Chiribella, D'Ariano, Perinotti
Phys Rev A 80, 022339 (2009)



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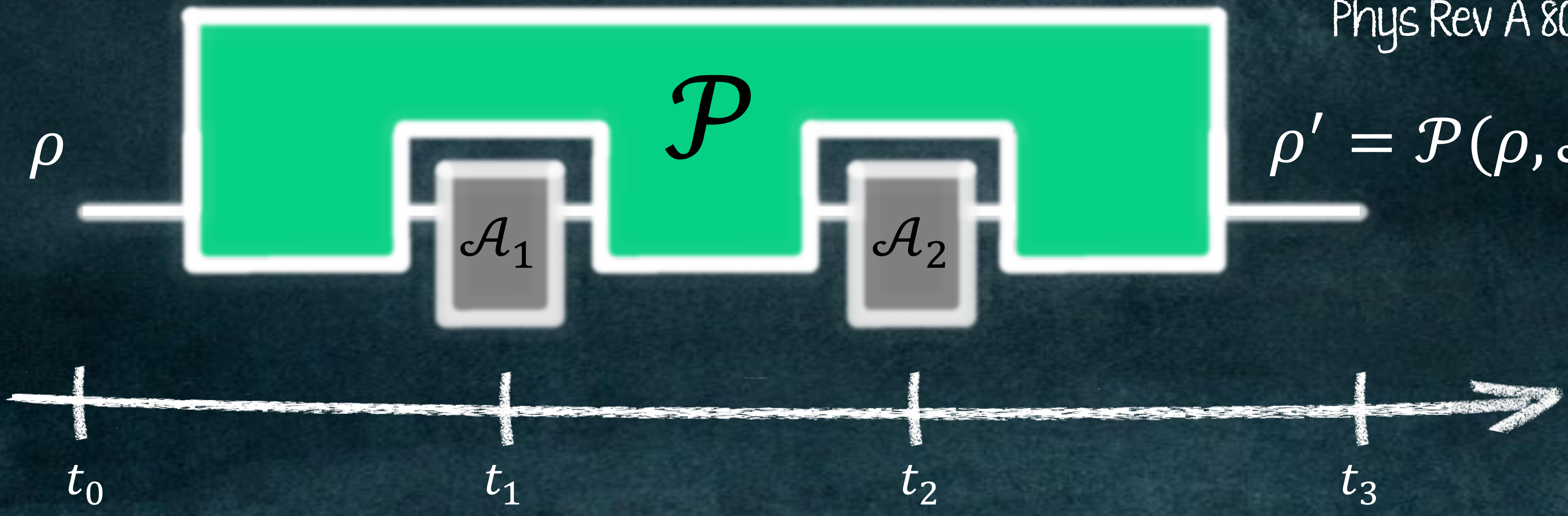

$$\rho' = \mathcal{P}(\rho, \mathcal{A}_1, \mathcal{A}_2, \ldots)$$

- **Process tensors** have the quantum comb structure

General multitime quantum processes

Chiribella, D'Ariano, Perinotti
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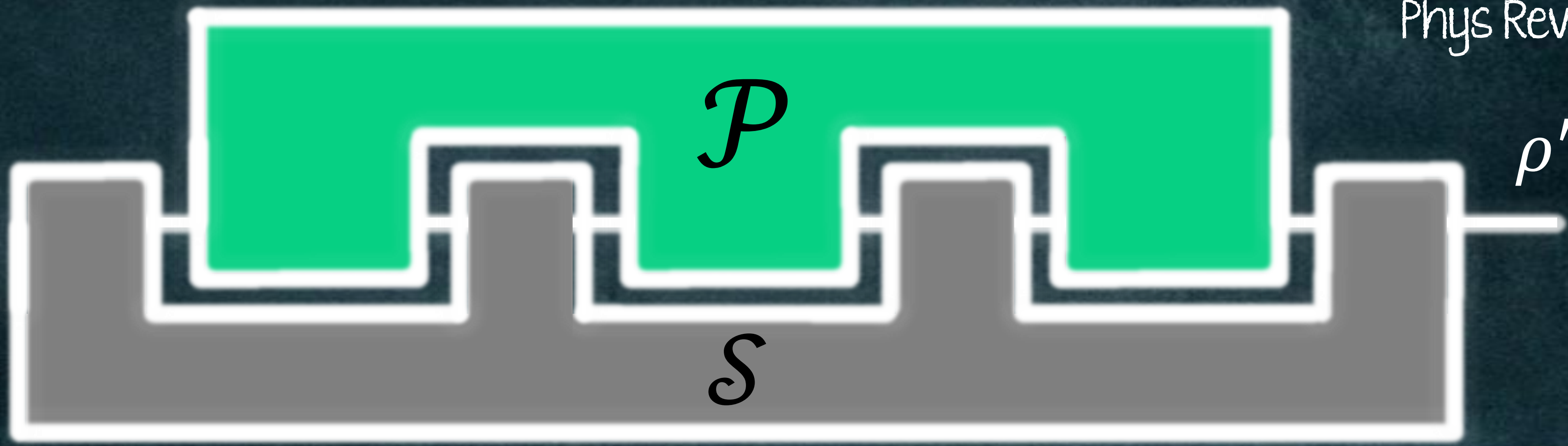
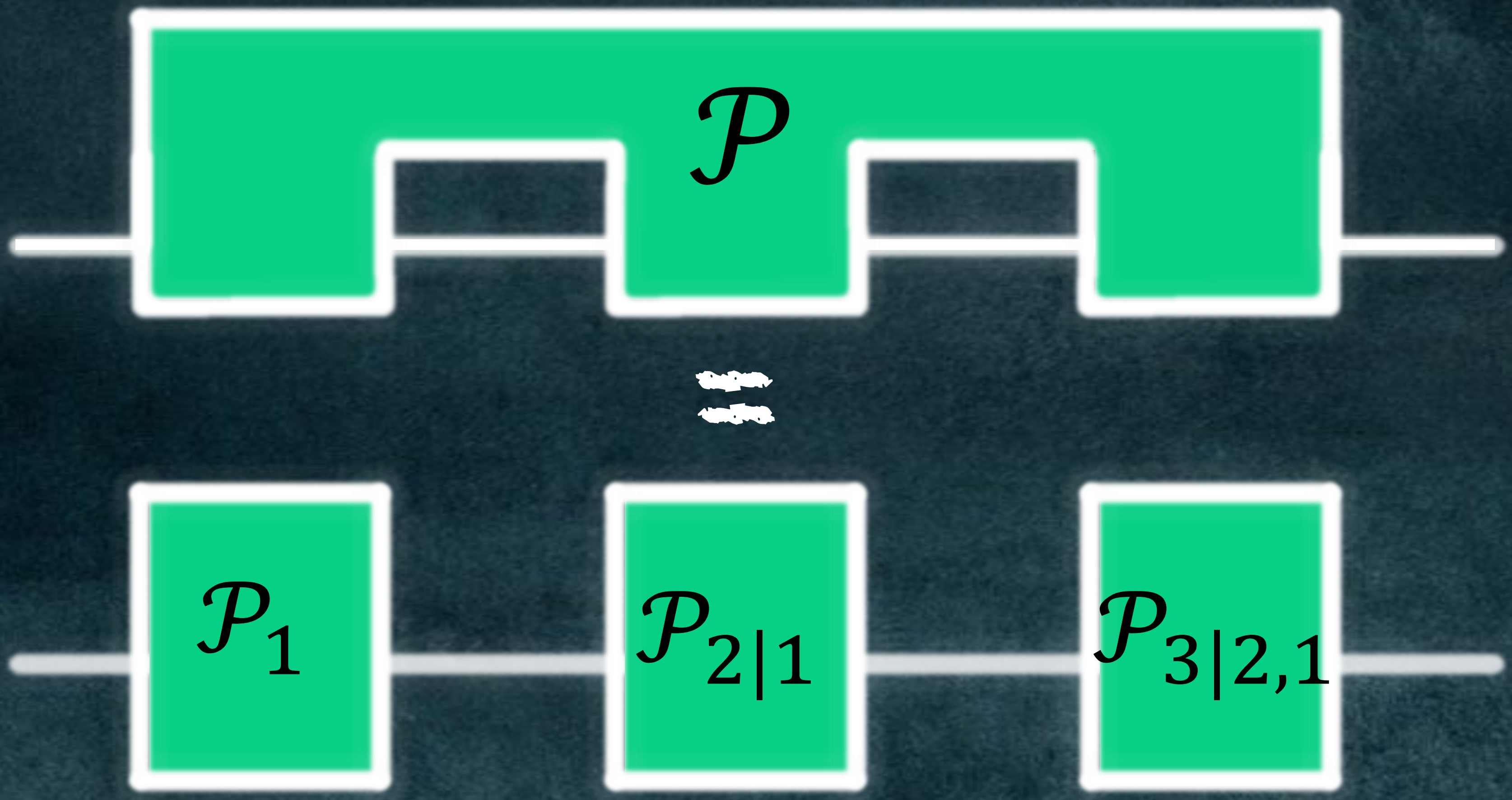


$\rho' = \mathcal{P}(\mathcal{S})$

- Process tensors have the quantum comb structure
- Control operations may also involve ancilas leading to quantum comb structure

Non-Markovian quantum processes

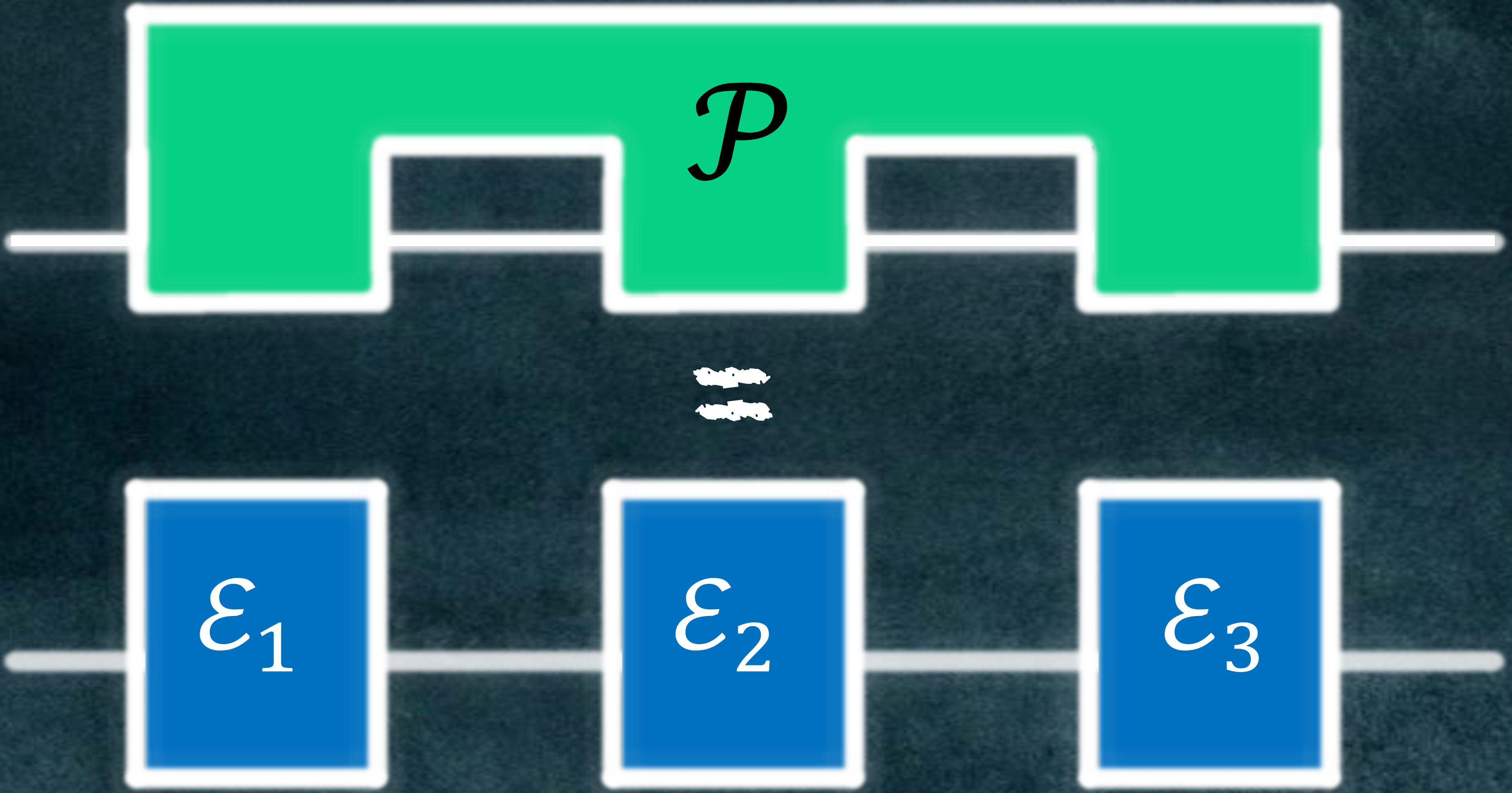
Modi et al, PRA, PRL (2018, 2019)



- Process tensors amount to sequences of quantum channels with memory

Markovian quantum processes

Modi et al, PRA, PRL (2018, 2019)

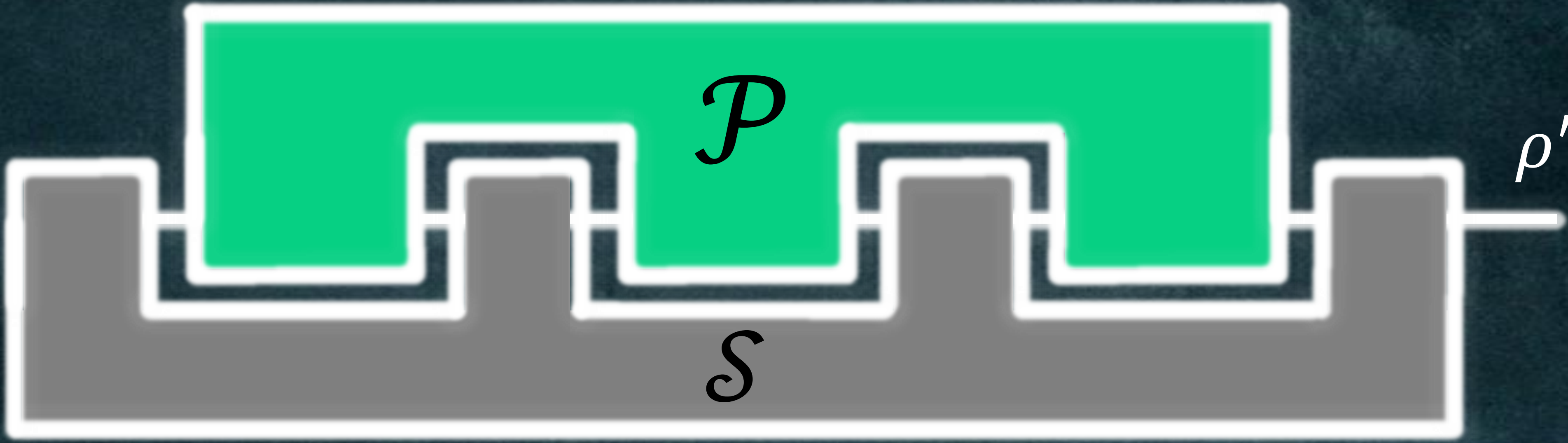


implies CP-divisibility

- Markovian processes are sequences of quantum channels without memory

Quantifying non-Markovianity

Zambon PRA 2024



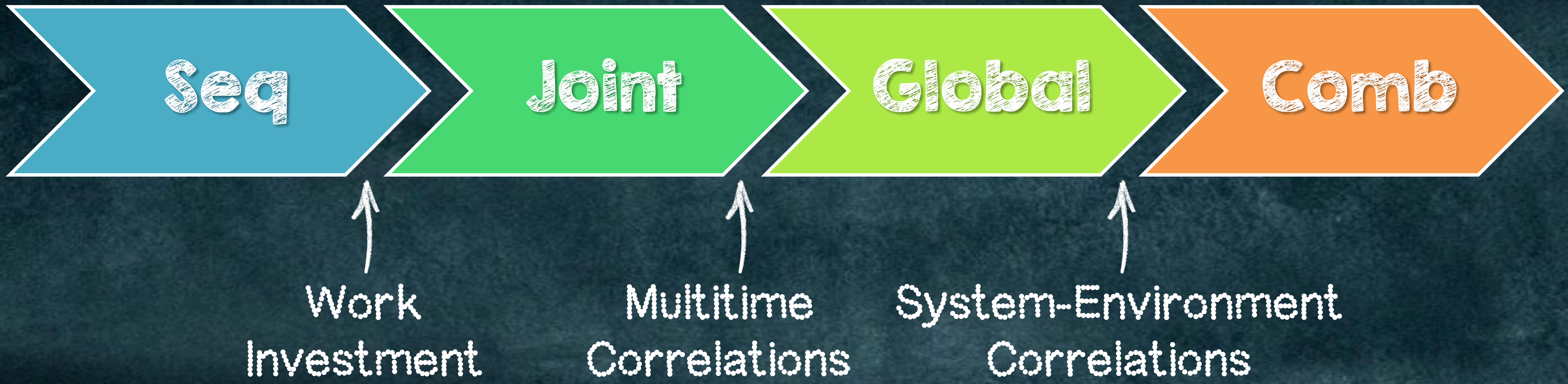
$\rho' = \mathcal{P}(\mathcal{S})$

$$N(\mathcal{P}) = \min_{\substack{\mathcal{M} \in \\ \text{Markov}}} \max_{\mathcal{S}} S(\mathcal{P}(\mathcal{S}) || \mathcal{M}(\mathcal{S}))$$

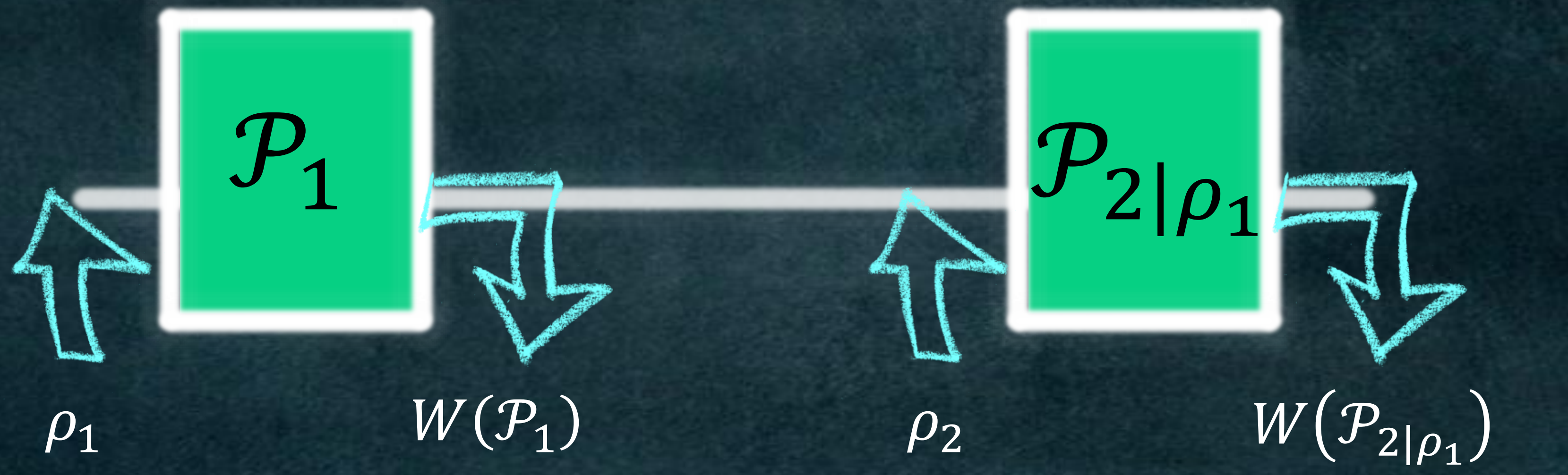
IS NON-MARKOVIANITY A THERMODYNAMIC RESOURCE?

Extracting work from processes

- We define 4 protocols for work extraction from multitime processes
- We identify 3 mechanisms yielding advantage from non-Markovianity

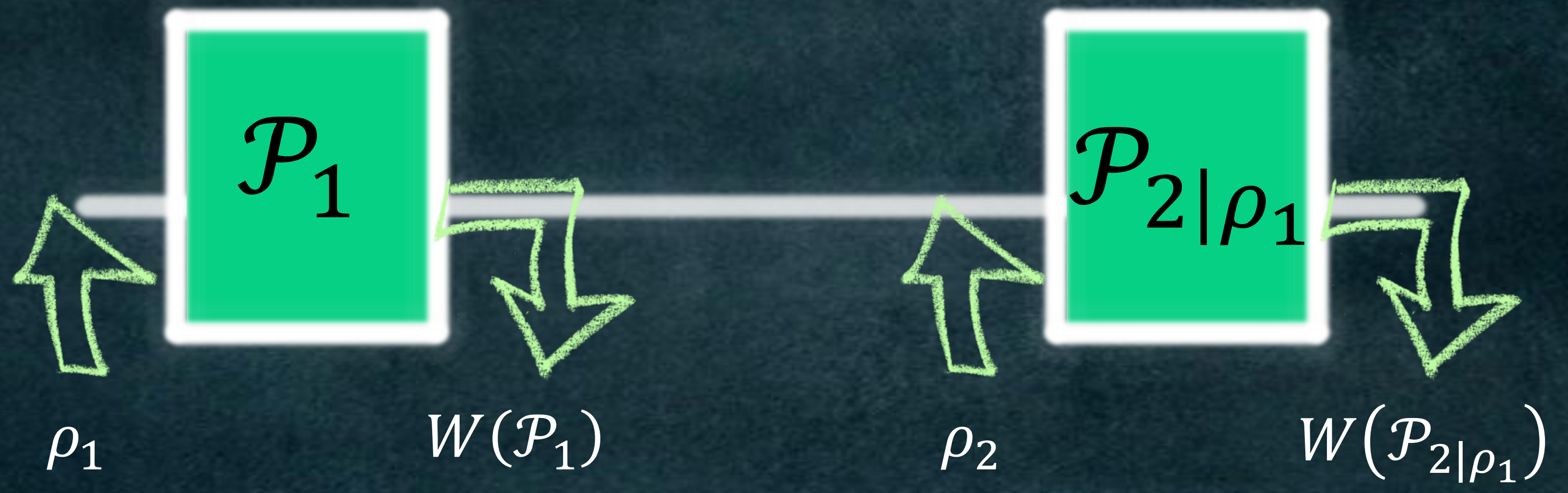


Sequential optimisation



- $W^{seq} = W(\mathcal{P}_1) + W\left(\mathcal{P}_{2|\rho_1}\right) + \cdots$
- Independently optimising the inputs for each channel in the sequence
- This is the best that we can get for a **Markovian** process tensor

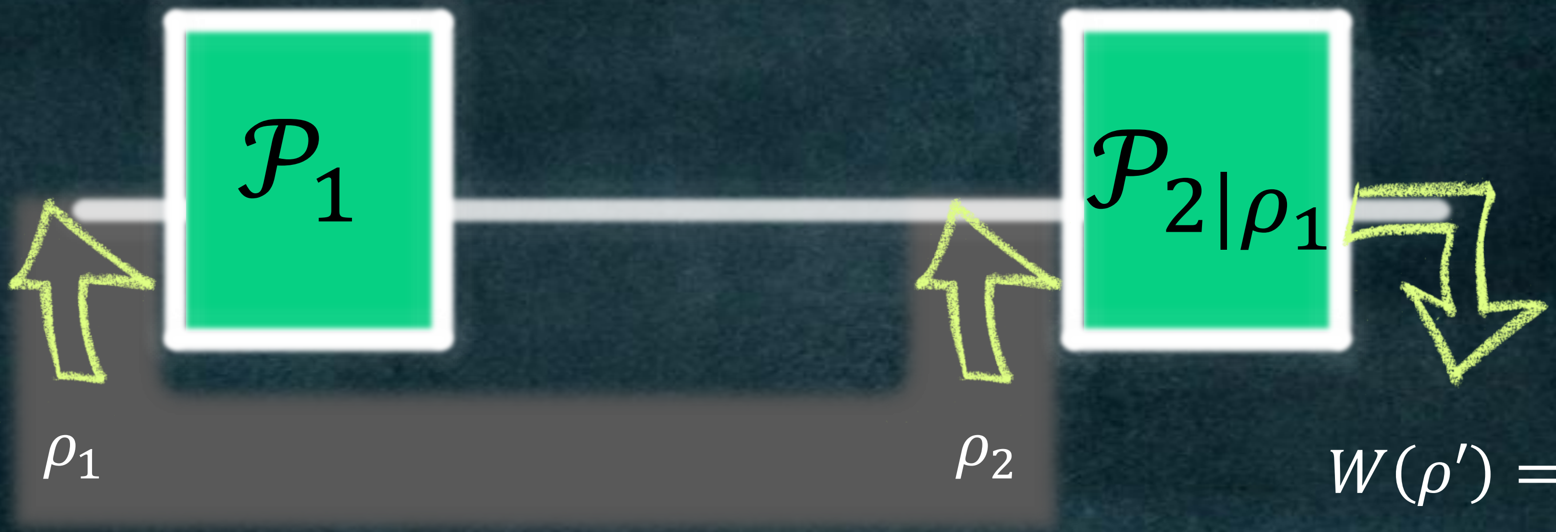
Joint optimisation



- $W^{joint} = \max_{\boldsymbol{r}} \sum_i W(\mathcal{P}_{i|\boldsymbol{r}})$, with $\boldsymbol{r} = (\rho_1, \rho_2, \ldots)$ sequence of input states
- Jointly optimising all the inputs (even if not optimal for individual nodes)
- **Work investment** (WI) may pay off leading to net extraction of more work

$$\Delta W^{\mathrm{WI}} := W^{joint} - W^{seq} \leq \propto \beta^{-1}[N(\mathcal{P})]^{1/4}$$

Global optimisation

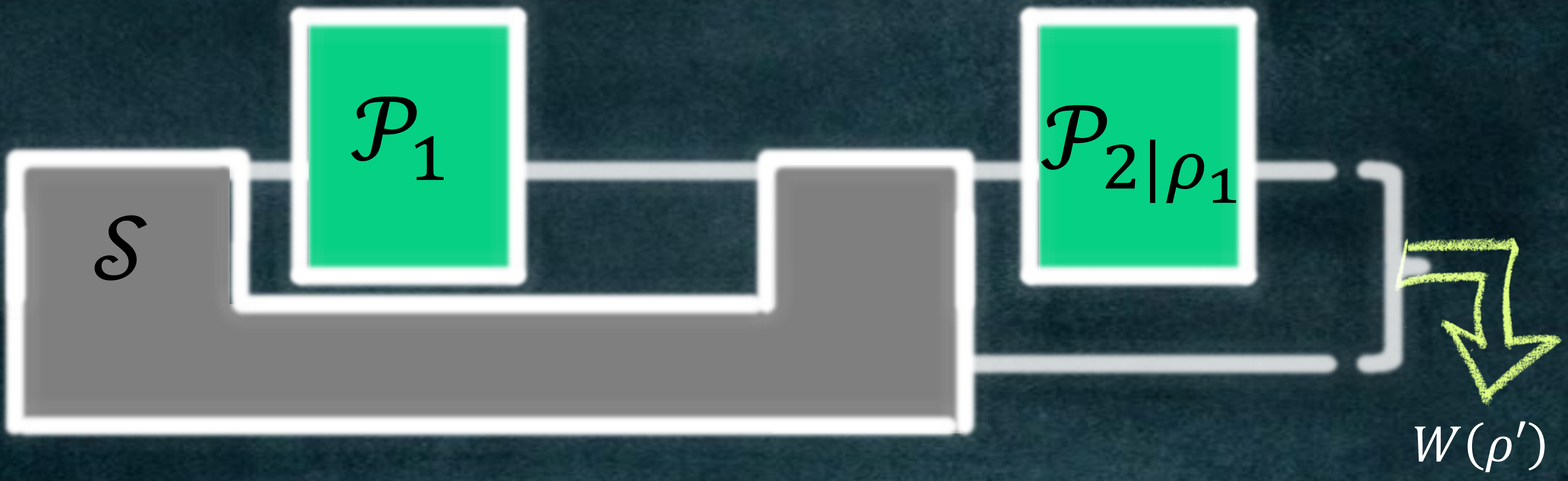


$W(\rho') = W\left(\mathcal{P}\left(\mathcal{S}_{\boldsymbol{\rho_1,\rho_2}}\right)\right)$

- $W^{global} = \max_{\boldsymbol{r}}\left[W\left(\mathcal{P}(\mathcal{S}_{\boldsymbol{r}})\right) - \sum_i W(\rho_i)\right]$
- with $\mathcal{S}_r$ the comb that inputs $\boldsymbol{r} = (\rho_1, \rho_2, \ldots)$ and stores all outputs till the end
- **Multitime correlations** (MTC) provide additional non-Markovian advantage

$$\Delta W^{\mathrm{MTC}} := W^{global} - W^{joint} \leq \beta^{-1} N(\mathcal{P})$$

Comb optimisation



- $W^{comb} = \max_{\mathcal{S}}[W(\mathcal{P}(\mathcal{S})) - W(\mathcal{S})]$
- where $\mathcal{S}$ can be dilated as a sequence of channels $\mathcal{E}_{SA}$ on system + ancilla
- System-environment correlations (SEC) may contribute the final power-up

$$\Delta W^{\mathrm{SEC}} := W^{comb} - W^{global} \leq \propto \beta^{-1}[N(\mathcal{P})]^{1/4}$$

Extracting work from a process $\mathcal{P}$

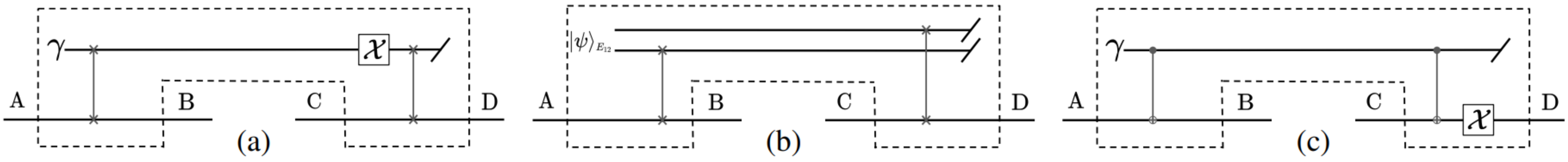
$$W^{seq} + \left(\Delta W^{\mathrm{WI}} + \Delta W^{\mathrm{MTC}} + \Delta W^{\mathrm{SEC}}\right) = W^{comb}$$

$$\leq \beta^{-1}\left[N(\mathcal{P}) + \propto [N(\mathcal{P})]^{1/4}\right]$$

Seq → Joint → Global → Comb

- Seq → Joint: Work Investment
- Joint → Global: Multitime Correlations
- Global → Comb: System-Environment Correlations

Simple examples of advantage



Seq → Joint → Global → Comb

Work Investment

Multitime Correlations

System-Environment Correlations


Seq
Joint
Global
Comb

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